## APPENDIX I.

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## APPENDIX II.

Retail Prices in Metropolitan Towns, 1901 to 1912.


* First 9 months.

Retail Prices in Metropolitan Towns, 1901 to 1912-contd.

| Town. | 1901. | 1902. | 1903. | 1904. | 1905. | 1906. | 1907. | 1908. | 1909. | 1910. | 1911. | 1912.* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Jam (Australian), per lb. |  |  |  |  |  |  |  |  |  |  |  |  |
|  | d. | d. | d. | d. | $d$. | d. | d. | d. | d. | d. | d. | d. |
| Sydney . . | 3.6 | 3.6 | 4.1 | 3.6 | 4.1 | 4.1 | 3.8 | 3.8 | 3.6 | 3.6 | 4.0 | 4.4 |
| Melbourne | 3.8 | 4.0 | 3.9 | 4.0 | 3.9 | 4.1 | 4.0 | 3.9 | 3.8 | 4.0 | 4.0 | 4.0 |
| Brisbane | 4.4 | 4.3 | 4.2 | 3.8 | 3.8 | 3.8 | 3.8 | 3.9 | 3.8 | 3.9 | 3.8 | 3.9 |
| Adelaide | 3.3 | 3.4 | 3.3 | 3.3 | 3.3 | 3.3 | 3.3 | 3.3 | 3.3 | 3.3 | 3.3 | 3.4 |
| Perth | 4.2 | 4.1 | 4.0 | 3.9 | 3.9 | 3.9 | 3.9 | 3.8 | 3.9 | 3.9 | 4.0 | 4.1 |
| Hobart | 4.1 | 4.1 | 4.1 | 4.2 | 4.1 | 4.1 | 4.1 | 4.1 | 4.1 | 4.3 | 4.3 | 4.2 |
| Oatmeal, per lb. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 22 | 2.5 | 2.5 | 2.2 | 2.2 | 2.3 | 2.6 | 2.8 | 2.6 | 26 | 28 | 3.0 |
| Melbourne | 2.2 | 2.6 | 2.1 | 1.6 | 1.7 | 2.1 | 2.2 | 2.4 | 2.1 | 2.4 | 2.2 | 2.7 |
| Brisbane | 2.6 | 2.4 | 2.6 | 2.3 | 2.2 | 2.4 | 2.4 | 2.7 | $\underline{9} .6$ | 2.6 | 2.7 | 2.8 |
| Adelaide | 2.1 | 2.5 | 2.4 | 1.7 | 1.9 | 2.1 | 2.1 | 2.3 | 2.0 | 2.0 | 2.2 | 2.8 |
| Perth | 2.5 | 2.7 | 2.7 | 2.2 | 2.1 | 2.1 | 2.2 | 2.4 | 2.3 | 2.2 | 2.2 | 2.9 |
| Hobart. | 2.0 | 2.2 | 2.3 | 1.6 | 1.7 | 2.0 | 2.1 | 2.3 | 2.2 | 2.1 | 2.0 | 2.8 |
| Raisins, per lb. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 6.2 | 6.9 | 7.0 | 6.0 | 5.9 | 5.3 | 4.8 | 7.2 | 5.9 | 6.5 | 6.0 | 6.2 |
| Melbourne | 7.7 | 6.4 | 6.3 | 5.5 | 6.0 | 6.3 | 6.4 | 6.4 | 6.2 | 6.5 | 6.5 | 6.3 |
| Brisbane | 7.4 | 6.2 | 5.4 | 5.4 | 5.3 | 5.3 | 6.0 | 6.2 | 6.6 | 6.6 | 6.6 | 6.4 |
| Adelaide | 6.5 | 6.3 | 6.3 | 6.1 | 6.1 | 6.6 | 6.3 | 6.3 | 6.3 | 6.5 | 6.5 | 6.2 |
| Perth | 7.5 | 8.2 | 7.5 | 7.7 | 7.4 | 7.5 | 7.6 | 7.4 | 7.4 | 7.2 | 7.4 | 6.4 |
| Hobart | 7.3 | 6.7 | 6.7 | 6.4 | 6.2 | 6.3 | 7.1 | 6.3 | 6.2 | 6.2 | 6.2 | 6.3 |

Currants, PER lb.

| Sydney | 6.6 | 5.6 | 5.6 | 5.2 | 5.7 | 5.9 | 6.2 | 6.6 | 6.6 | 6.6 | 6.9 | 7.3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 6.6 | 5.6 | 5.3 | 5.3 | 5.4 | 5.6 | 6.2 | 6.7 | 6.6 | 6.8 | 6.8 | 7.1 |
| Brisbane | 7.8 | 6.5 | 5.8 | 5.7 | 5.6 | 5.5 | 6.5 | 6.7 | 7.0 | 7.0 | 7.0 | 7.2 |
| Adelaide | 7.1 | 6.8 | 8.1 | 5.7 | 5.7 | 6.3 | 6.4 | 6.8 | 7.2 | 7.2 | 7.2 | 6.7 |
| Perth | 6.5 | 6.0 | 5.5 | 6.0 | 6.1 | 6.2 | 6.7 | 6.6 | 6.6 | 6.7 | 7.0 | 7.0 |
| Hobart | 7.1 | 6.1 | 5.7 | 5.2 | 5.5 | 5.5 | 6.6 | 7.0 | 7.1 | 7.1 | 7.4 | 7.7 |

Staroh. per lb.

| Sydney . . | 3.8 | 3.5 | 4.8 | 5.5 | 5.0 | 5.4 | 5.5 | 5.5 | 5.5 | 6.0 | 6.0 | 5.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 4.8 | 5.3 | 5.1 | 4.9 | 4.7 | 4.9 | 4.8 | 4.8 | 5.0 | 5.0 | 5.0 | 5.3 |
| Brisbane | 5.6 | 5.5 | 5.3 | 5.3 | 5.3 | 5.2 | 5.4 | 5.3 | 5.3 | 5.3 | 5.4 | 5.5 |
| Adelaide | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 5.5 |
| Perth | 4.8 | 5.9 | 5.9 | 5.9 | 5.9 | 5.9 | 5.9 | 5.9 | 5.9 | 5.9 | 5.9 | 5.9 |
| Hobart | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 |

Blue, per dozen squares.

| Sydney | 8.9 | 8.9 | 8.9 | 8.9 | 8.9 | 8.9 | 8.9 | 8.9 | 8.9 | 8.9 | 8.9 | 8.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 5.1 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.2 | 6.4 | 6.3 | 6.4 | 6.6 |
| Brisbane | 8.6 | 8.8 | 8.7 | 7.7 | 7.9 | 7.3 | 7.4 | 7.6 | 7.9 | 8.0 | 7.9 | 8.3 |
| Adelaide | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 9.3 |
| Perth | 11.1 | 11.1 | 11.1 | 11.1 | 11.1 | 11.1 | 11.1 | 11.1 | 10.6 | 10.6 | 10.6 | 10.9 |
| Hobart | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 9.2 | 8.5 | 9.2 | 9.2 | 9.2 | 9.0 |

Candles, per lb.

| Sydney | 5.6 | 5.6 | 5.6 | 6.6 | 6.6 | 8.6 | 6.6 | 6.6 | 6.6 | 6.6 | 6.6 | 6.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 7.1 | 6.4 | 6.5 | 6.4 | 6.3 | 6.3 | 6.5 | 6.6 | 6.6 | 6.6 | 6.6 | 6.4 |
| Brisbane | 7.1 | 6.8 | 6.2 | 6.4 | 6.4 | 6.4 | 6.6 | 6.5 | 6.4 | 6.7 | 6.6 | 6.6 |
| Adelaide | 6.8 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.2 | 7.2 | 7.2 | 7.1 |
| Perth | 6.8 | 7.3 | 7.3 | 7.3 | 6.9 | 6.9 | 7.2 | 6.9 | 6.4 | 6.2 | 6.4 | 7.4 |
| Hobart | 4.6 | 4.6 | 4.6 | 4.6 | 4.6 | 4.6 | 4.8 | 5.3 | 5.4 | 5.4 | 5.4 | 5.9 |

SOAP, PER Lb.

| Sydney | 2.2 | 2.2 | 2.6 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 3.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 3.0 | 2.9 | 3.0 | 3.0 | 2.9 | 2.9 | 3.1 | 3.0 | 3.0 | 3.1 | 3.1 | 3.6 |
| Brisbane | 2.6 | 2.7 | 2.5 | 2.3 | 2.5 | 2.3 | 2.5 | 2.5 | 2.5 | 2.5 | 2.6 | 2.4 |
| Adelaide | 2.4 | 2.4 | 2.5 | 2.6 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.8 | 2.8 | 2.6 |
| Perth | 2.7 | 2.6 | 2.7 | 2.7 | 2.7 | 2.8 | 3.0 | 3.0 | 2.8 | 2.8 | 3.0 | 2.9 |
| Hobart. | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.7 | 2.7 | 2.7 | 3.2 |

* First 9 months.

Retail Prices in Metropolitan Towns, 1901 to 1912-contd.

| Potatoes, per 14 Lbs. |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sydney | ${ }^{8 .}{ }^{\text {d }}$ d. ${ }^{\text {a }}$ | 8. $\begin{gathered}\text { d. } \\ 11.3\end{gathered}$ | s. $\begin{gathered}d \\ 8.3\end{gathered}$ | s. ${ }^{\text {d }}$ d. ${ }^{\text {a }}$ | $\left.{ }^{\text {8. }}{ }^{\text {d }} 3.8\right\|_{1}{ }^{8}$ | 8. $\bar{d}$. | 8. $\begin{gathered}\text { d. } \\ 6.8\end{gathered}$ | 8. $\begin{gathered}\text { d. } \\ 10.9\end{gathered}$ | s. ${ }^{\text {d }}$ d | ${ }^{8 .}$ d. | 811.3 | $\left[\begin{array}{cc} s . d \\ 1 & 8.9 \end{array}\right.$ |
| Melbourne | 8.5 | 10.5 | 8.3 | 6.11 | $1 \quad 0.91$ | 1.7 | 7.5 | 9.6 | 8.9 | 11.3 | 10.0 | 181.9 |
| Brisbane | 10.81 | 10.1 | 7.6 | 6.81 | 12.31 | 3.6 | 10.7 | 12.1 | $1 \quad 1.8$ | 12.71 | 14.9 | 198 |
| Adelaide | 110.9 | 1.3 | 8.8 | 8.91 | 11.41 | 1.9 | 9.1 | 10.7 | 11.7 | 11.9 | 11.4 | 14.3 |
| Perth | 1.5 .8 | $1 \quad 5.5$ | 13.311 | 4.41 | 17.71 | 7.71 | 3.7 | 14.5 | $1 \quad 4.0$ | $1 \quad 5.5$ | 28.3 | 19.8 |
| Hobart | 10.4 | 9.9 | 7.9 | 6.31 | $1 \quad 1.211$ | 4.7 | 6.2 | 8.1 | 10.6 | $1 \quad 0.3$ | 11.5 | 13.8 |
| ONIONS, PER LB. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 1.4 | 0.8 | 0.6 | 0.5 | 2.01 | 1.0 | 0.6 | 1.2 | 1.1 | 0.7 | 0.6 | 2.1 |
| Melbourne | 1.1 | 0.8 0.9 | 0.8 | 0.7 | 1.2 | 1.1 | 0.7 | 1.5 | 1.2 | 1.1 | 0.8 | 2.0 |
| Brisbane | 1.7 | 1.3 | 1.0 | 0.9 | 2.3 | 1.2 | 1.0 | 1.6 | 1.4 | 0.9 | 0.9 | 2.1 |
| Adelaide | 1.9 | 1.2 | 1.0 | 1.0 | 2.2 | 1.3 | 1.0 | 1.4 | 1.5 | 1.2 | 1.3 | 2.3 |
| Perth | 2.0 | 2.0 | 1.5 | 1.7 | 2.5 | 1.7 | 1.7 | 1.8 | 1.9 | 1.6 | 1.6 | 2.4 |
| Hobart | 1.3 | 1.3 | 1.4 | 1.1 | 2.1 | 1.1 | 1.1 | 1.6 | 1.5 | 1.5 | 1.5 | 2.5 |
| Kerosene, per gallon. |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 10.1 |  |  |  |  |  |  |  |
| Syduey | 10.1 | 10.1 | 10.1 | 10.8 | 10.1 | 10.1 | 10.1 | 11.8 | 11.8 | 11.8 | 11.8 | 100.8 |
| Mielbourne | 10.0 | 9.6 | 10.5 | 10.5 | $10.6{ }^{\text {' }}$ | 11.5 | 11.7 | 11.0 | 11.3 | 11.2 | 11.3 | 110.3 |
| Brisbane | 18.9 | 13.7 | 1.1 .41 | 1.6 | 11.71 | 1.91 | 13.3 | $1 \begin{array}{ll}1 & 2.1\end{array}$ | $1 \begin{array}{ll}1 & 2.2\end{array}$ | $1 \quad 2.0$ | 11.9 | 10.5 |
| Adelaide | 1.4 .5 | 10.1 | 1. 0.71 | 0.7 | $1 \quad 0.21$ | 0.31 | 1.3 | $1 \begin{array}{ll}1 & 0.7\end{array}$ | $1 \quad 1.0$ | 11.0 | $1 \quad 0.8$ | 1. 2.0 |
| Perth | 1. 0.2 | 11.6 | 1.2 | 11.5 | $11.1{ }^{\text {1 }}$ | 11.51 | 0.6 | 11.6 | 11.6 | 11.7 | 10.3 | 10.5 |
| Hobart | 14.3 | 12.4 | $1 \begin{array}{ll}1 & 1.3\end{array}$ | 0.6 | 0.71 | 1.3 ! | 1.8 | 12.3 | 11.9 | 2.2 | 1.8 | 12.7 |
| Milk, per quart. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydnoy | 4.0 | 4.6 | 4.5 | 3.8 | 4.0 | 4.01 | 4.3 | 5.0 | 4.8 | 4.5 | 4.4 | 5.0 |
| Melbourne | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.2 | 4.6 | 4.1 | 4.0 | 4.2 | 4.8 |
| Brisbane | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 4.9 | 4.9 | 4.8 |
| Adelaide | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.4 | 5.0 | 5.0 | 5.1 | 5.9 | 5.9 |
| Perth | 5.9 | 5.9 | 6.4 | 5.9 | 5.9 | 5.9 | 5.9 | 6.4 | 0.4 | 6.4 | 6.4 | 6.9 |
| Hobart. | 4.1 | 4.1 | 4.3 | 4.3 | 4.4 | 4.4 | 4.6 | 4.4 | 4.6 | 4.6 | 4.8 | 5.0 |

Butter, per lb.

| Sydney | 10.2 | 14.4 | 2.2 |  | 10.2 | 1 | 1.0 | 1 | 1.2 | 1 | 1.0 | 1 | 3.4 | 1 | 2.4 |  | 1.5 | 1 | 1.7 | 1 | 3.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 12.8 | $1 \quad 5.4$ | $1 \quad 2.5$ | 1 | 0.2 | 1 | 2.0 | 1 | 1.7 | 1 | 2.3 | 1 | 4.3 | 1 | 2.9 | 1 | 2.4 | 1 | 1.7 | 1 | 4.2 |
| Brisbane | 11.9 | 13.6 | 12.0 |  | 9.9 |  | 11.8 | 1 | 0.0 |  | 11.7 | 1 | 3.0 | 1 | 1.3 | 1 | 0.7 | 1 | 1.0 | 1 | 3.0 |
| Adelaide | 14.0 | $1 \begin{array}{ll}1 & 5.2\end{array}$ | $1 \quad 1.0$ |  | 11.2 | 1 | 1.2 | 1 | 1.2 | 1 | 0.7 | 1 | 3.6 | 1 | 2.2 | 1 | 1.3 | 1 | 9.0 | 1 | 5.8 |
| Perth | 1.6 .9 | 17.1 | $1 \begin{array}{ll}1 & 5.0 \\ 1 & 0.7\end{array}$ | 1 | 3.7 | 1 | 4.0 | 1 | 4.0 | 1 | 3.7 | 1 | 4.8 | 1 | 3.7 | 1 | 3.3 | 1 | 3.4 | 1 | 4.9 |
| Hobart | 11.0 | 12.1 | $1 \begin{array}{ll}1 & 0.7\end{array}$ |  | 10.8 | 1 | 0.7 | 1 | 0.6 | 1 | 0.8 | 1. | 2.9 | 1 | 2.1 | 1 | 1.9 | 1 | 1.2 | 1 | 4.2 |

Charse, per hb.

| Sydney | 7.8 | 10.0 | 9.5 | 7.6 | 9.5 | 0.0 | 10.3 | 10.0 | 10.7 | 9.8 | 9.5 | 11.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 9.2 | 10.4 | 10.4 | 10.5 | 9.9 | 10.4 | 9.9 | 10.7 | 10.3 | 10.5 | 9.9 | 11.1 |
| Brisbane | 10.0 | 10.8 | 10.9 | 9.2 | 9.9 | 8.8 | 9.8 | 11.0 | 10.9 | 10.4 | 10.6 | 11.2 |
| Adelaide | 10.9 | 10.4 | 10.2 | 10.4 | 10.4 | 10.1 | 10.3 | 10.4 | 10.5 | 10.7 | 10.7 | 11.5 |
| Perth | 10.3 | 10.8 | $1 \quad 0.3$ | 11.5 | 11.3 | 10.4 | 10.4 | 11.2 | 10.7 | 10.4 | 10.2 | 11.9 |
| Hobart | 8.2 | 9.3 | 9.4 | 8.9 | 8.6 | 8.1 | 9.2 | 10.2 | 9.7 | 9.3 | 9.3 | 11.5 |

Egas, per dozen.

| Sydney | $1 \quad 3.7$ | 0.8 | $1 \begin{array}{ll}1 & 6.8\end{array}$ | 12.3 | 1 | 0.6 | 1 | 1.7 | 1 | 3.0 | 1 | 5.2 | 1 | 5.4 | 1 | 5.0 | 1 | 5.0 |  | 8.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 1.4 .1 | $1 \quad 4.2$ | $1 \quad 2.8$ | 3.0 | 1 | 2.5 | 1 | 3.8 | 1 | 4.3 | 1 | 3.9 | 1 | 4.2 | 1 | 5.4 | 1 | 5.2 | 1 | 6.6 |
| Brisbane | 10.3 | 10.7 | $1 \begin{array}{ll}1 & 0.1\end{array}$ | 9.5 |  | 9.9 |  | 9.7 |  | 11.3 | 1 | 1.9 | 1 | 1.6 | 1 | 2.8 | 1 | 3.4 | 1 | 6.9 |
| Adelaide | 10.3 | $1 \begin{array}{ll}1 & 0.7\end{array}$ | $1 \begin{array}{ll}1 & 1.7\end{array}$ | 0.2 | 1 | 0.0 |  | 11.5 | 1 | 0.1 | 1 | 1.4 | 1 | 0.8 | 1 | 1.2 | 1 | 1.4 | 1 | 3.9 |
| Perth | 1.8 .5 | $2 \quad 0.2$ | $2 \begin{array}{ll}2 & 1.3\end{array}$ | 17 | 1 | 8.5 | 1 | 9.3 | 1 | 8.6 |  | 10.2 | 1 | 8.2 | 1 | 8.3 | 1 | 8.4 | 1 | 10.4 |
| Hobart | 111 | $1 \quad 2.3$ | 14.3 | $1 \begin{array}{ll}1 & 1.2\end{array}$ | 1 | 2.5 | 1 | 4.0 | 1 | 1.7 | 1 | 3.4 | 1 | 4.8 | 1 | 1.0 | 1 | 2.8 | 1 | 5.6 |

Bacon (Middle Cut), per lb.

| Syduey | 0.1 | 10.8 | 10.5 | 10.2 | 9.3 | 9.8 | 10.3 |  | 11.7 | 1 | 0.0 |  | 10.3 | 10.0 |  | 10.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 11.1 | 11.6 | 10.1 | $1 \quad 0.1$ | 10.7 | 10.7 | 10.7 |  | 11.4 |  | 11.6 |  | 11.6 | 10.9 |  | 11.8 |
| Brisbane | 8.4 | 8.9 | 10.7 | 8.6 | 7.1 | 7.6 | 8.9 |  | 10.0 |  | 10.2 |  | 9.7 | 9.3 |  | 10.3 |
| Adelaide | 10.7 | 11.2 | 11.7 | 11.1 | 10.0 | 10.5 | 10.5 |  | 11.0 |  | 11.4 |  | 11.1 | 11.0 |  | 10.9 |
| Perth | 11.8 | 11.2 | 11.9 | $1 \quad 1.7$ | 11.2 | 0.6 | 0.1 | 1 | 0.2 | 1 | 0.4 | 1 | 0.2 | 0.2 | 1 | 0.3 |
| Hobart. | 10.1 | 10.2 | 11.1 | 9.4 | 8.4 | 8.6 | 9.9 |  | 10.8 |  | 10.7 |  | 10.4 | 10.0 |  | 10.7 |

[^0]Retail Prices in Metropoliten Towns, 1901 to 1912-contd.


Beef, Fresh, Sirloin, per lb.

| Sydney | 6.0 | 7.4 | 6.3 | 5.7 | 5.7 | 5.7 | 5.8 | 5.8 | 5.7 | 5.7 | 5.7 | 5.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 6.1 | 6.8 | 6.3 | 5.9 | 5.8 | 5.5 | 5.8 | 5.9 | 5.7 | 5.5 | 5.1 | 6.2 |
| Brisbane | 4.8 | 5.0 | 5.0 | 4.5 | 4.3 | 4.6 | 4.5 | 4.7 | 4.2 | 4.5 | 4.2 | 4.4 |
| Adelaide | 6.4 | 6.0 | 6.0 | 5.7 | 5.8 | 5.8 | 5.7 | 5.7 | 5.9 | 5.6 | 5.7 | 5.8 |
| Perth | 5.8 | 6.0 | 5.9 | 5.8 | 6.3 | 6.5 | 6.3 | 6.1 | 6.1 | 6.5 | 6.8 | 7.7 |
| Hobart | 6.1 | 6.9 | 6.7 | 6.5 | 6.5 | 6.4 | 6.3 | 6.4 | 6.4 | 6.4 | 6.4 | 6.4 |

Beef, Fresh, Rib, per hb.

| Sydney | 4.7 | 6.1 | 5.0 | 4.5 | 4.5 | 4.5 | 4.6 | 4.6 | 4.5 | 4.5 | 4.5 | 4.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 5.2 | 5.8 | 5.1 | 4.7 | 4.6 | 4.6 | 4.9 | 5.0 | 4.8 | 4.6 | 4.2 | 5.0 |
| Brisbane | 4.1 | 4.3 | 4.1 | 3.9 | 3.7 | 4.1 | 3.9 | 4.1 | 3.4 | 3.7 | 3.7 | 3.3 |
| Adelaide | 5.4 | 5.0 | 5.0 | 4.8 | 4.9 | 4.9 | 5.0 | 5.0 | 4.9 | 4.7 | 4.7 | 4.8 |
| Perth | 5.5 | 5.8 | 6.1 | 5.4 | 5.7 | 6.1 | 5.7 | 5.6 | 5.7 | 6.0 | 6.3 | 6.6 |
| Hobart. | 5.6 | 6.4 | 6.3 | 5.8 | 6.0 | 6.0 | 5.7 | 5.9 | 60 | 59 | 5.6 | 5.4 |

Beef, Fresh, Flank, per lb.

| Sydney . . | 3.6 | 4.7 | 3.9 | 3.4 | 3.4 | 3.4 | 3.5 | 3.5 | 3.4 | 3.4 | 3.4 | 3.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 4.5 | 5.2 | 4.5 | 4.0 | 4.0 | 3.9 | 4.2 | 4.4 | 4.2 | 3.9 | 3.6 | 4.0 |
| Brisbane | 5.5 | 5.5 | 5.5 | 5.5 | 5.5 | 5.5 | 5.5 | 5.5 | 4.6 | 3.7 | 4.6 | 3.4 |
| Adelaide | 4.1 | 4.1 | 4.0 | 4.0 | 4.0 | 4.0 | 4.1 | 4.1 | 3.9 | 3.8 | 3.8 | 4.2 |
| Perth | 5.9 | 6.9 | 7.2 | 6.3 | 6.8 | 7.0 | 6.0 | 6.5 | 6.6 | 6.9 | 7.3 | 6.1 |
| Hobart | 4.0 | 4.6 | 4.5 | 4.2 | 4.2 | 4.0 | 4.1 | 4.1 | 4.2 | 3.9 | 3.6 | 3.9 |

Beef, Fresh, Shin, per lb.

| Sydney | 3.3 | 4.3 | 3.5 | 3.0 | 3.0 | 3.0 | 3.2 | 3.2 | 3.0 | 3.0 | 3.0 | 3.4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 3.9 | 4.4 | 3.8 | 3.6 | 3.5 | 3.4 | 3.8 | 3.9 | 3.7 | 3.4 | 3.1 | 3.6 |
| Brisbane | 3.5 | 3.3 | 3.1 | 3.1 | 3.1 | 3.1 | 3.0 | 2.9 | 2.4 | 2.7 | 2.7 | 3.0 |
| Adelaide | 3.9 | 3.6 | 3.5 | 3.4 | 3.5 | 3.5 | 3.5 | 3.5 | 3.9 | 3.4 | 3.4 | 3.7 |
| Perth | 4.9 | 5.7 | 5.7 | 4.9 | 4.9 | 4.9 | 4.9 | 4.9 | 4.9 | 4.9 | 4.9 | 5.9 |
| Hobart | 4.0 | 4.3 | 4.5 | 4.1 | 4.6 | 4.3 | 4.1 | 4.1 | 4.3 | 4.0 | 4.0 | 4.5 |

Steak, RUMp, per lb.

| Sydney | 7.3 | 8.7 | 8.1 | 6.7 | 6.7 | 6.7 | 7.0 | 7.0 | 6.7 | 6.7 | 6.7 | 8.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 8.0 | 8.8 | 8.4 | 8.2 | 8.2 | 8.0 | 8.4 | 8.3 | 8.0 | 7.5 | 6.9 | 8.7 |
| Brisbane | 5.3 | 6.1 | 6.3 | 5.7 | 5.6 | 6.5 | 6.3 | 6.6 | 6.2 | 6.3 | 6.1 | 6.4 |
| Adelaide | 7.8 | 7.6 | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 | 8.3 | 8.0 | 8.0 | 8.1 |
| Perth | 9.7 | 10.7 | 11.2 | 10.2 | 10.2 | 10.4 | 10.4 | 10.4 | 10.7 | 10.4 | 10.5 | 11.8 |
| Hobart | 8.0 | 8.6 | 8.6 | 8.1 | 8.4 | 8.3 | 8.4 | 8.0 | 8.4 | 8.1 | 8.1 | 8.4 |

Steak, Shoulder, per lb.

| Sydney | 4.1 | 5.0 | 4.3 | 3.4 | 3.4 | 3.4 | 3.6 | 3.6 | 3.4 | 3.4 | 3.4 | 4.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 4.6 | 5.2 | 4.5 | 4.3 | 4.2 | 4.2 | 4.5 | 4.4 | 4.2 | 4.1 | 3.9 | 4.4 |
| Brisbane | 3.7 | 4.2 | 4.3 | 3.7 | 3.6 | 4.1 | 4.2 | 4.5 | 3.8 | 3.7 | 3.5 | 3.8 |
| Adelaide | 5.4 | 5.0 | 5.0 | 4.9 | 4.9 | 5.0 | 4.5 | 4.4 | 4.5 | 4.1 | 4.4 | 4.5 |
| Perth | 5.6 | 6.3 | 7.2 | 6.3 | 7.1 | 7.5 | 7.0 | 6.5 | 6.9 | 7.3 | 7.3 | 7.2 |
| Hobart. | 5.1 | 5.6 | 5.6 | 5.4 | 5.3 | 5.1 | 5.3 | 5.2 | 5.4 | 5.2 | 5.1 | 5.3 |

* First 9 months of 1912.

Retail Prices in Metropoliten Towns, 1901 to 1912 -contd.


Beef (Corned), Brisket With bone, per lb.

| Sydney | 3.6 | 5.3 | 4.7 | 3.4 | $3.4{ }^{1}$ | 3.4 | 3.6 | 3.5 | 3.4 | 3.4 | 3.4 | 3.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nielbourne | 3.1 | 3.7 | 3.0 | 3.1 | 3.0 | 3.0 | 3.3 | 3.1 | 3.0 | 2.7 | 2.4 | 3.1 |
| Brisbane | 3.4 | 4.0 | 4.0 | 3.4 | 3.3 | 3.4 | 3.4 | 3.7 | 3.3 | 3.1 | 3.1 | 2.9 |
| Adelaide | 4.1 | 3.9 | 3.9 | 3.6 | 3.9 | 3.9 | 3.6 | 3.6 | 3.7 | 3.6 | 3.6 | 3.6 |
| Perth | 4.0 | 4.7 | 5.2 | 4.4 | 4.71 | 4.7 | 4.5 | 4.5 | 4.5 | 4.7 | 4.7 | 4.6 |
| Hobart | 4.2 | 4.7 | 4.9 | 4.3 | 4.5 | 4.3 | 4.3 | 4.3 | 4.5 | 4.2 | 4.0 | 3.5 |
| Beef (Corned), Brisket without bond, Per lb. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 4.7 | 6.3 | 5.8 | 4.4 | 4.4 | 4.4 | 4.6 | 4.6 | 4.4 | 4.4 | 4.4 | 3.9 |
| Nielbourne | 4.3 | 4.9 | 4.2 | 4.1 | 4.0 | 3.9 | 4.2 | 4.1 | 3.9 | 3.7 | 3.6 | 4.1 |
| Brisbane | 4.0 | 4.5 | 4.5 | 3.7 | 4.0 | 4.2 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 3.9 |
| Adelaide | 5.5 | 5.3 | 5.3 | 5.0 | 5.3 | 5.3 | 4.7 | 4.7 | 4.7 | 4.7 | 4.2 | 4.5 |
| Perth | 5.3 | 0.2 | 6.5 | 5.7 | 6.2 | 6.2 | 6.0 | 5.7 | 5.8 | 6.2 | 6.5 | 6.0 |
| Hobart. | 5.0 | 5.4 | 5.5 | 5.1 | 5.1 | 5.0 | 5.2 | 5.1 | 5.2 | 5.0 | 5.0 | 4.4 |

Mution, Leg, per lb.

| Syduey | 4.2 | 5.0 | 4.6 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 3.7 | 4.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 4.0 | 4.3 | 4.2 | 4.4 | 4.1 | 3.9 | 4.1 | 3.8 | 3.5 | 3.6 | 3.4 | 4.2 |
| Brisbane | 4.7 | 5.1 | 5.1 | 4.6 | 4.6 | 4.7 | 4.6 | 4.7 | 3.9 | 3.8 | 3.9 | 4.0 |
| Adelaide | 5.0 | 4.6 | 4.6 | 4.4 | 4.4 | 4.2 | 3.8 | 3.8 | 4.1 | 3.8 | 3.8 | 4.4 |
| Perth | 7.0 | 7.7 | 7.2 | 7.5 | 7.6 | 7.2 | 7.3 | 7.5 | 7.1 | 7.3 | 7.6 | 7.9 |
| Hoburt | 4.9 | 5.4 | 5.2 | 5.4 | 5.2 | 5.3 | 5.4 | 5.2 | 5.6 | 5.5 | 4.7 | 5.3 |

Mutton, Shoulder, per lb.


Mutton, Loin, per lb.

| Sydney | 4.1 | 4.7 | 4.5 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.8 | 3.7 | 4.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mielbourne | 3.9 | 4.4 | 4.2 | 4.4 | 4.1 | 4.0 | 4.1 | 3.8 | 3.7 | 3.8 | 3.5 | 4.2 |
| Bristane | 4.6 | 5.0 | 5.0 | 4.6 | 4.5 | 4.7 | $4 . \overline{5}$ | 4.8 | 4.6 | 4.0 | 4.1 | 4.8 |
| Adelaide | 4.6 | 4.4 | 4.5 | 4.3 | 4.3 | 4.1 | 3.8 | 3.8 | 4.0 | 4.0 | 4.0 | 4.5 |
| Perth | 0.0 | 0.7 | 6.3 | 0.5 | 6.7 | 6.3 | 6.4 | 0.5 | 6.2 | 6.4 | 6.6 | $7 \cdot$ |
| Hobart | 5.5 | 5.9 | 6.0 | 5.6 | 5.6 ! | 5.8 | 5.7 | 5.7 | 6.2 | 6.2 | 5.3 | 5.1 |

Mution, Neok, per Lb.

*First 9 months of 1912.

Retail Prices in Metropolitan Towns, 1901 to 1912-contd..

| TOWN. | 1901. | 1902. | 1903. | 1904. | 1905. | 1906. | 1907. | 1908. | 1909. | 1910. | 1911. | $1912 . *$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Chops, Loin, per lib.

|  | ${ }_{\text {d. }}$. | ${ }^{\text {d. }}$ | d. | d. | ${ }_{\text {d }}$ | d. | d. | d. | ${ }^{\text {d }}$ | d. | d. | $d$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sydney | 4.4 | 5.5 | 4.8 | 4.3 | 4.3 | 4.3 | 4.3 | 4.3 | 4.3 | 4.1 | 4.0 | 5.4 |
| Melbourne | 5.0 | 5.5 | 5.4 | 5.3 | 5.0 | 4.8 | 5.0 | 4.7 | 4.3 | 4.5 | 3.9 | 5.2 |
| Brisbane | 4.9 | 5.4 | 5.7 | 5.0 | 4.8 | 4.9 | 4.9 | 5.1 | 4.9 | 4.3 | 4.3 | 5.1 |
| Adelaide | 5.5 | 5.2 | 4.9 | 4.7 | 4.9 | 4.9 | 4.7 | 4.7 | 4.7 | 4.4 | 4.4 | 5.1 |
| Perth | 6.3 | 7.3 | 7.0 | 7.2 | 7.5 | 7.0 | 7.1 | 6.9 | 6.9 | 7.1 | 7.3 | 8.0 |
| Hobart | 5.7 | 6.1 | 6.0 | 5.8 | 5.8 | 5.8 | 6.1 | 5.8 | 5.8 | 5.8 | 5.8 | 6.1 |

Chops, Leg, per ib.

| Sydney | 4.1 | 5.4 | 4.7 | 4.2 | 4.2 | 4.2 | 4.2 | 4.2 | 4.2 | 4.1 | 3.9 | 4.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 4.5 | 4.9 | 4.8 | 4.8 | 4.5 | 4.3 | 4.3 | 4.2 | 3.9 | 4.0 | 3.8 | 4.7 |
| Brisbane | 4.9 | 5.4 | 5.4 | 5.0 | 4.9 | 4.9 | 4.9 | 5.1 | 4.9 | 4.3 | 4.3 | 5.1 |
| Adelaide | 5.7 | 5.4 | 5.1 | 4.9 | 5.1 | 5.1 | 4.9 | 4.9 | 4.9 | 4.6 | 4.6 | 5.3 |
| Perth | 6.5 | 7.8 | 7.3 | 7.5 | 7.7 | 7.3 | 7.6 | 7.1 | 7.1 | 7.4 | 7.4 | 8.1 |
| Hobart | 5.2 | 5.7 | 5.7 | 5.7 | 5.7 | 5.7 | 5.7 | 5.7 | 5.7 | 5.7 | 5.3 | 6.0 |

Chops, NeOk, per lb.


Pork, Leg, per lib.

| Sydney | 7.0 | 6.7 | 6.7 | 6.7 | 6.7 | 6.7 | 7.4 | 7.9 | 8.1 | 8.6 | 8.6 | 7.8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 5.4 | 6.4 | 6.2 | 5.7 | 5.2 | 5.0 | 5.5 | 5.5 | 6.2 | 5.7 | 5.2 | 6.4 |
| Brisbane | 5.8 | 6.4 | 7.0 | 6.0 | 5.8 | 5.9 | 5.4 | 5.4 | 5.8 | 5.8 | 6.3 | 7.0 |
| Adelaide | 7.3 | 7.3 | 7.0 | 6.9 | 7.0 | 6.6 | 5.9 | 6.4 | 6.4 | 6.4 | 64 | 7.1 |
| Perth | 7.9 | 8.1 | 7.9 | 8.1 | 8.4 | 8.4 | 8.5 | 8.7 | 8.6 | 8.1 | 8.1 | 8.9 |
| Hobart. | 6.4 | 6.2 | 6.3 | 6.2 | 6.0 | 6.1 | 6.5 | 6.3 | 6.0 | 6.5 | 6.1 | 6.2 |

Pork, Loin, per lb.

| Sydney | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.8 | 7.3 | 7.5 | 8.0 | 8.0 | 8.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 6.4 | 7.4 | 7.0 | 6.6 | 6.0 | 6.0 | 6.4 | 6.6 | 7.0 | 6.5 | 6.0 | 7.1 |
| Brisbane | 6.3 | 6.9 | 7.5 | 7.0 | 6.4 | 6.6 | 6.4 | 6.6 | 6.8 | 6.8 | 7.3 | 7.2 |
| Adelaide | 7.7 | 7.7 | 7.3 | 7.2 | 7.3 | 6.9 | 6.7 | 7.2 | 7.2 | 7.2 | 7.2 | 7.0 |
| Perth | 6.9 | 7.2 | 6.9 | 7.2 | 7.4 | 7.4 | 7.9 | 7.8 | 7.7 | 7.2 | 7.2 | 9.0 |
| Hobart | 6.6 | 6.6 | 6.9 | 6.8 | 6.6 | 6.6 | 6.6 | 6.9 | 6.6 | 6.6 | 6.7 | 6.5 |

Pork, Belly, per lb.

| Sydney . . | 5.5 | 5.9 | 5.9 | 5.9 | 5.9 | 5.9 | 6.5 | 7.1 | 7.3 | 7.5 | 7.5 | 7.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 6.2 | 7.3 | 7.0 | 6.6 | 6.1 | 6.0 | 6.4 | 6.6 | 70 | 6.5 | 6.1 | 6.9 |
| Brisbane | 5.0 | 5.5 | 6.0 | 5.7 | 5.5 | 5.1 | 5.2 | 5.4 | 5.7 | 5.5 | 6.5 | 6.0 |
| Adelaide | 7.6 | 7.6 | 7.2 | 7.1 | 7.2 | 6.9 | 6.6 | 7.1 | 7.1 | 7.1 | 7.1 | 7.8 |
| Perth | 7.0 | 7.3 | 7.0 | 7.3 | 7.5 | 7.5 | 7.9 | 7.8 | 7.6 | 7.1 | 7.1 | 8.1 |
| Hobart | 6.7 | 7.0 | 6.7 | 6.9 | 6.6 | 6.9 | 6.9 | 6.7 | 6.5 | 6.6 | 6.7 | 6.5 |

Pork, Chops, per lib.

| Sydaey | 7.0 | 6.7 | 6.7 | 6.7 | 6.7 | 6.7 | 7.4 | 7.9 | 8.1 | 8.6 | 8.6 | 8.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 7.0 | 8.1 | 7.6 | 7.2 | 6.6 | 6.4 | 7.0 | 7.2 | 7.6 | 7.1 | 6.6 | 7.3 |
| Brisbane | 6.7 | 7.3 | 8.1 | 7.0 | 6.7 | 6.8 | 6.6 | 6.8 | 7.0 | 7.2 | 7.7 | 7.9 |
| Adelaide | 8.0 | 8.0 | 7.8 | 7.8 | 7.8 | 7.3 | 6.8 | 7.3 | 7.3 | 7.3 | 7.3 | 8.0 |
| Perth | 8.6 | 8.7 | 8.5 | 8.7 | 9.0 | 9.0 | -9.1 | 9.4 | 9.3 | 8.7 | 8.7 | 9.6 |
| Hobart | 7.4 | 7.5 | 7.4 | 6.8 | 6.7 | 6.8 | 7.1 | 6.9 | 7.2 | 6.8 | 6.8 | 7.0 |

* First 9 months of 1912.


## APPENDIX III.

Current Retail Prices in Metropolitan and Country Towns, 1912.*

| Town. | Bread | Flour | Tea | Coffee | Sugar per lb. | Rice per lb. | $\begin{gathered} \text { Sago } \\ \text { per 1b. } \end{gathered}$ | Jam per lb. | Oatmeal per lb. | Raisins per lb. | Currants per 1b. | Starch <br> per Ib. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | d. | s. $\quad d$. | s. d. | s. $\quad d$. | d. | d. | d. | d. | d. | d. | $d$. | $d$. |
| Sydney | 3.3 | 210.4 | 14.0 | 1.6 .2 | 2.8 | 2.8 | 3.0 | 4.4 | 3.0 | 6.2 | 7.3 | 5.6 |
| Newcastle | 3.0 | 211.0 | 15.4 | $1 \begin{array}{ll}1 & 5.6\end{array}$ | 2.9 | 3.2 | 3.2 | 4.4 | 2.9 | 7.0 | 7.5 | 5.6 |
| Broken Hill | 3.5 | 211.7 | 16.2 | $\begin{array}{ll}1 & 7.8\end{array}$ | 3.3 | 4.0 | 4.1 | 4.5 | 3.5 | 7.1 | 7.4 | 6.9 |
| Goulburia . | 3.2 | $\begin{array}{ll}2 & 9.9\end{array}$ | $\begin{array}{ll}1 & 5.9\end{array}$ | $\begin{array}{ll}1 & 6.6\end{array}$ | 3.0 | 3.1 | 3.7 | 4.6 | 3.1 | 7.6 | 7.7 | 6.0 |
| Bathurst . . | 3.5 | $2 \quad 6.7$ | 15.8 | 16.0 | 3.1 | 3.0 | 3.4 | 4.9 | 3.0 | 7.2 | 7.1 | 6.1 |
| Melbourne | 3.0 | 126.7 | $1 \quad 2.7$ | $1 \quad 6.6$ | 3.0 | 2.8 | 2.9 | 4.0 | 2.7 | 6.3 | 7.1 | 5.3 |
| Ballarat | 3.1 | $2 \quad 6.4$ | $1 \quad 3.0$ | $1 \quad 5.9$ | 3.0 | 2.9 | 2.9 | 3.6 | 2.7 | 5.8 | 6.7 | 5.4 |
| Bendigo | 3.2 | $\begin{array}{ll}2 & 6.0\end{array}$ | $1 \quad 2.2$ | 110.1 | 3.2 | 2.9 | 3.0 | 3.5 | 2.8 | 5.8 | 7.0 | 5.3 |
| Geelong . . | 3.2 | $\begin{array}{ll}2 & 9.4 \\ 2 & 9.4\end{array}$ | $1 \begin{array}{ll}1 & 2.6\end{array}$ | 11 5.2 <br> 1  | 3.0 | 2.6 | 3.0 | 3.7 | 2.9 | 6.2 | 7.5 | 5.1 |
| Warrnambool | 3.3 | 288.3 | $1 \begin{array}{ll}1 & 3.2\end{array}$ | $1 \quad 6.1$ | 2.9 | 2.7 | 3.0 | 3.8 | 2.8 | 6.3 | 0.9 | 5.8 |
| Brisbane | 3.5 | $3 \quad 1.7$ | $1 \quad 4.3$ | 17.1 | 3.0 | 2.6 | 2.8 | 3.9 | 2.8 | 6.4 | 7.2 | 5.5 |
| Toowoomba | 3.5 | $3 \quad 4.0$ | $1 \begin{array}{ll}1 & 0.0\end{array}$ | $1 \begin{array}{ll}1 & 5.9\end{array}$ | 3.1 | 3.0 | 3.2 | 4.8 | 3.0 | 7.1 | 7.7 | 0.0 |
| Rockhampt'n Charters | 3.8 | $3 \quad 0.9$ | $1 \quad 6.0$ | $1 \quad 6.6$ | 2.8 | 2.8 | 3.0 | 4.4 | 2.9 | 0.2 | 7.0 | 6.0 |
| Trowers | 4.5 | $3 \quad 7.2$ | $1 \quad 7.6$ | 17.8 | 3.5 | 3.7 | 4.0 | 4.8 | 3.3 | 6.8 | 7.7 | 6.0 |
| Warwick . | 3.7 | $3 \quad 3.9$ | $1 \begin{array}{ll}1 & 6.1\end{array}$ | 10.0 | 3.1 | 3.0 | 3.4 | 4.8 | 3.1 | 8.2 | 7.6 | 0.0 |
| Adelaide | 3.5 | 20.1 | 14.4 | 1.5 .9 | 2.9 | 3.3 | 3.2 | 3.4 | 2.8 | 6.2 | 6.7 | 5.5 |
| Kadina Moonta \& |  |  |  |  |  |  |  |  |  |  |  |  |
| Wallaroo .. | 3.3 | 26.1 | 113.4 | $1 \begin{array}{ll}1 & 6.2\end{array}$ | 2.9 | 3.1 | 3.1 | 3.5 | 2.8 | 6.6 | 6.6 | 5.9 |
| Port Pirie | 3.0 | 29.1 | $1 \quad 6.0$ | $1 \quad 7.9$ | 3.3 | 3.4 | 3.7 | 3.9 | 3.1 | 7.1 | 7.5 | 6.2 |
| Mt. Gambier | 3.0 | $\begin{array}{ll}2 & 9.9\end{array}$ | $1 \begin{array}{ll}1 & 4.2\end{array}$ | $1 \begin{array}{ll}1 & 7.7\end{array}$ | 3.0 | 3.0 | 3.2 | 4.0 | 2.9 | 6.6 | 7.1 | 5.7 |
| Petersburg | 3.4 | 211.2 | $1 \quad 5.6$ | 17.5 | 3.2 | 3.5 | 3.0 | 4.2 | 3.0 | 7.4 | 6.9 | 6.2 |
| Perth and Fremantle | 3.5 | 28.4 | $1 \begin{array}{ll}1 & 3.8\end{array}$ | 17.2 | 3.0 | 2.9 | 3.0 | 4.1 | 2.9 | 6.4 | 7.0 | 5.9 |
| Kalgoorlie \& Boulder | 5.0 | $3 \begin{array}{ll}3 & 3.1\end{array}$ | $1 \begin{array}{ll}1 & 7.3\end{array}$ | $1 \quad 9.4$ | 3.8 | 3.9 | 4.0 | 5.2 | 3.4 | 9.3 | 8.1 | 7.0 |
| Mid. Junction |  |  |  |  |  |  |  |  |  |  |  |  |
| \& Guildford | 3.5 | $2 \quad 9.0$ | $1 \begin{array}{ll}1 & 3.8 \\ 1 & 3.7\end{array}$ | $1 \begin{array}{ll}1 & 6.0\end{array}$ | 3.0 | 2.9 | 3.0 | 4.0 | 3.0 | 6.0 | 7.3 | 6.0 |
| 73unbury . . | 3.5 | 210.6 | $1 \begin{array}{ll}1 & 3.7\end{array}$ | $1 \begin{array}{ll}1 & 6.0 \\ 1 & 3.0\end{array}$ | 3.2 | 3.1 | 3.1 | 4.2 | 2.9 | 6.7 | 7.3 | 6.1 |
| Geraliton | 4.0 | 211.1 | $1 \quad 5.4$ | 13.4 | 3.3 | 3.0 | 4.0 | 4.3 | 3.0 | 7.8 | 7.9 | 6.0 |
| Hobart | 3.5 | $\begin{array}{ll}3 & 0.5\end{array}$ | $1 \begin{array}{ll}1 & 3.4\end{array}$ | $1 \begin{array}{ll}1 & 0.0\end{array}$ | 3.0 | 3.0 | 3.2 | 4.2 | 2.8 | 6.3 | 7.7 | 6.0 |
| Launceston | 3.3 | 287.4 | $1 \quad 2.5$ | 1.5 | 2.7 | 2.9 | 2.8 | 3.9 | 2.5 | 6.1 | 7.1 | 5.4 |
| Zeehath | 3.7 | 211.3 | $1 \begin{array}{ll}1 & 4.2\end{array}$ | $1 \begin{array}{ll}1 & 6.1\end{array}$ | 3.0 | 3.0 | 3.2 | 4.2 | 2.9 | 6.6 | 7.1 | 5.5 |
| Beacons fleld | 3.2 | 210.6 | $1 \begin{array}{ll}1 & 2.8\end{array}$ | $1 \begin{array}{ll}1 & 5.8\end{array}$ | 3.0 | 3.0 | 3.4 | 4.3 | 2.8 | 7.5 | 7.1 | 5.9 |
| Queenstown | 3.7 | $\begin{array}{ll}3 & 0.2\end{array}$ | 15.6 | $1 \begin{array}{ll}1 & 4.7\end{array}$ | 2.9 | 3.0 | 3.1 | 4.0 | 2.7 | 7.6 | 7.8 | 6.7 |
| Weighted Average | 3.3 | 29.4 | $1 \begin{array}{ll}1 & 3.8\end{array}$ | $1 \quad 6.4$ | 3.0 | 2.9 | 3.0 | 4.1 | 2.8 | 6.4 | 7.2 | 5.6 |

* Average prices for first 9 months only.

Current Retail Prices in Metropolitan and Country Towns, 1912.*-contd.

| Town. | Blue. dz. sq. | Candle <br> per lb. | Soap <br> per lb. | Potatoes. 14 lbs . | Onions <br> per lb. | Kerosene gallon | Milk quart | Butter | Cheese <br> per lb. | Eggs <br> per dz | Bacon Middle <br> per lb. | Bacon <br> Shouder. per lb. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | s. d. | d. | $d$. | 8. d. | d. | $s . \quad d$. | d. | 8. d. |  | 8. $\quad$ d | s. d. | d. |
| Sydney |  | 6.6 | 3.0 | 16.9 |  | 10.8 | 5.0 | 13.6 | 11.7 | 18.1 | 10.5 | 6.8 |
| Newcastle | 10.0 | 7.0 | 3.6 | $1 \quad 7.5$ | 2.5 | 12.1 | 4.3 | 13.7 | 11.7 | 188.3 | 10.8 | 9.5 |
| Broken Hill | 1.0 .7 | 8.5 | 3.2 | 18.8 | 2.8 | 19.6 | 6.2 | 18.3 | 0.5 | $1 \quad 6.4$ | 11.3 | 9.1 |
| Goulburn | 11.2 | 6.4 | 3.2 | $1 \begin{array}{ll}1 & 3.7\end{array}$ | 2.6 | 13.2 | 4.9 | $1 \begin{array}{ll}1 & 3.9\end{array}$ | 11.5 | $1 \quad 5.9$ | 10.6 | 7.8 |
| Bathurst | 11.2 | 7.3 | 2.9 | $1 \quad 2.4$ | 2.3 | $1 \quad 5.3$ | 4.4 | $1 \begin{array}{ll}1 & 3.7\end{array}$ | 11.7 | $1 \quad 5.9$ | 11.4 | 9.0 |
| Melbourne | 6.6 | 6.4 | 3.6 | $1 \quad 1.9$ | 2.0 | $1 \quad 0.3$ | 4.8 | 14.2 | 11.1 | 16.6 | 11.8 | 6.8 |
| Ballarat | 6.3 | 5.9 | 2.9 | 11.9 | 2.0 | $1 \quad 1.5$ | 3.9 | $1 \quad 3.1$ | 10.9 | $1 \quad 2.9$ | 11.5 | 6.9 |
| Bendigo | 6.4 | 6.6 | 3.1 | 13.3 | 2.1 | 12.5 | 4.8 | 13.6 | 11.3 | $1 \begin{array}{ll}1 & 3.7\end{array}$ | 10.2 | 6.2 |
| Geelong | 6.3 | 6.0 | 3.0 | 1.1 .6 | 1.9 | 12.6 | 4.5 | $1 \quad 3.4$ | 10.7 | $1 \begin{array}{ll}1 & 4.4\end{array}$ | 11.3 | 6.9 |
| Warrnambool | 7.1 | 6.2 | 2.9 | 1.1 .3 | 1.7 | 11.3 | 4.0 | $1 \quad 4.2$ | 10.6 | $1 \quad 3.2$ | 1.0 .4 | 6.8 |
| Brisbane | 8.3 | 6.6 | 2.4 | 19.9 | 2.1 | $1 \quad 0.5$ | 4.8 | $1 \quad 3.0$ | 11.2 | $1 \quad 6.9$ | 10.3 | 7.1 |
| Toowoomba | 10.0 | 7.2 | 3.0 | 110.6 | 2.4 | 14.9 | 4.2 | $1 \quad 3.4$ | 11.0 | 150.7 | 9.9 | 7.5 |
| Rockhampt'n | 9.5 | 6.9 | 2.5 | $1 \quad 9.8$ | 2.3 | 11.5 | 4.9 | $1 \quad 2.5$ | 11.8 | 17.3 | 9.3 | 7.6 |
| Charters Towers | 10.3 | 7.8 | 2.8 | 24.3 | 2.9 | $1 \quad 6.3$ | 4.8 | 15.5 | 0.7 | 19.3 | 11.5 | 8.8 |
| Warwick . | 11.6 | 7.3 | 4.9 | 19.6 | 2.7 | $1 \quad 6.0$ | 4.2 | 13.6 | 10.9 | $1 \quad 4.6$ | 10.0 | 8.4 |
| Adelaide | 9.3 | 7.1 | 2.6 | 1. 4.3 | 2.3 | 12.0 | 5.9 | 15.8 | 11.5 | 13.9 | 10.9 | 6.5 |
| Kadina Moonta \& |  |  |  |  |  |  |  |  |  |  |  |  |
| Wallaroo . | 9.6 | 7.3 | 3.1 | $1 \quad 4.8$ | 2.5 | 13.2 | 6.0 | $1 \quad 5.2$ | 11.6 | 12.0 | 10.7 | 0.2 |
| Port Pirie | 11.6 | 8.5 | 2.5 | $1 \quad 5.3$ | 2.6 | 14.1 | 5.8 | $1 \begin{array}{ll}1 & 5.9\end{array}$ | 10.8 | 12.8 | 11.7 | 93 |
| Mt. Gambier | 11.4 | 7.8 | 3.0 | $1 \begin{array}{ll}1 & 1.9\end{array}$ | 2.2 | $1 \quad 38$ | 3.4 | 12.5 | 10.1 | $1 \begin{array}{ll}1 & 0.1\end{array}$ | 10.6 | 8.1 |
| Petersburg | 11.7 | 7.9 | 3.5 | 15.4 | 2.8 | 15.0 | 4.4 | 14.4 | 11.6 | 12.0 | 11.5 | 9.8 |
| Perth and Fremantle | 10.9 | 7.4 | 2.9 | $1 \quad 9.8$ | 2.4 | 10.5 | 6.9 | $1 \quad 4.9$ | 11.9 | 110.4 | $1 \quad 0.3$ | 7.8 |
| Kalgooriie \& Boulder | 10.0 | 9.5 | 4.3 | $2 \quad 3.6$ | 3.3 | 19.7 | 9.0 | $1 \begin{array}{ll}1 & 7.3\end{array}$ |  |  | $\begin{array}{ll}1 & \cdot 1.7\end{array}$ | 0.6 |
| Mid. Junction |  |  |  |  |  |  |  |  |  |  |  |  |
| \& Guildford | 10.8 | 8.2 | 3.1 | $2 \quad 0.0$ | 2.7 | $1 \quad 0.5$ | 6.0 | 1 - 5.5 |  | 110.1 | $1 \begin{array}{ll}1 & 0.3\end{array}$ | 8.7 |
| Bunbury . . | 11.2 | 8.5 | 2.7 | 110.7 | 2.8 | $1 \begin{array}{ll}1 & 1.0\end{array}$ | 5.5 | $1 \quad 5.7$ | 10.1 | 188.9 | $1 \quad 0.0$ | 8.4 |
| Geraldton | $1 \quad 0.0$ | 8.7 | 2.7 | 21.0 | 2.4 | $1 \quad 2.3$ | 6.0 | $1 \begin{array}{ll}1 & 7.0\end{array}$ | $1 \quad 0.9$ | 111.7 | $1 \begin{array}{ll}1 & 1.3\end{array}$ | 9.6 |
| Hobart | 9.0 | 5.9 | 3.2 | $1 \begin{array}{ll}1 & 3.8\end{array}$ | 2.5 | 1.2 .7 | 5.0 | 14.2 | 11.5 | $1 \quad 5.6$ | 10.7 | 6.8 |
| Launceston | 7.3 | 5.7 | 28 | $1 \begin{array}{ll}1 & 3.7\end{array}$ | 2.2 | $1 \begin{array}{ll}1 & 3.4\end{array}$ | 4.6 | $1 \quad 3.3$ | 10.5 | $1 \begin{array}{ll}1 & 4.0\end{array}$ | 10.0 | 7.7 |
| Zeehan | 9.1 | 7.0 | 3.1 | $1 \quad 7.3$ | 2.3 | $1 \quad 3.2$ | 5.6 | $1 \quad 5.0$ | 11.5 | $\begin{array}{ll}1 & 7.2\end{array}$ | 10.0 | 7.8 |
| Beaconsfleld | 9.4 | 7.2 | 4.2 | 14.0 | 2.5 | $1 \quad 3.4$ | 4.8 | 14.1 | 10.9 | 14.8 | 9.7 | 8.2 |
| Queenstown | 8.7 | 7.7 | 3.3 | 16.4 | 2.1 | 15.4 | 5.6 | $1 \quad 4.3$ | 10.6 | $1 \quad 7.2$ | 10.0 | 8.2 |
| Weighted Average | 8.4 | 6.7 | 3.1 | 15.4 | 2.2 | $1 \begin{array}{ll}1 & 1.4\end{array}$ | 5.1 | 14.2 | 11.5 | 16.9 | 11.1 | 7.1 |

* Average prices for first 9 months only.

Current Retail Prices in Metropolitan and Country Towns, 1912.*-contd.

| Town. | Ham per lb. | Beef Fresh Sirloin <br> per lb. | Beef Fresh Rib <br> per lb. | Beef Fresh Flank <br> per 1b. | Beef Fresh Shin <br> per lb. | Steak Rump per lb. | Steak sh'lder per lb. | Steak Buttock <br> per lb. | Beef Co'n'd round <br> per lb. | Beef Co'n'd brisket with bone per lb. | Beef <br> Co'n'd <br> brisket <br> with- <br> out <br> bone <br> per lb. | Mutt'n Leg <br> per lb. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 8. d. | $d$. | d. | $d$. | d. | 8. d. | d. | d. | d. | $d$. | d. | d. |
| Sydney | 11.5 | 5.9 | 4.7 | 3.9 | 3.4 | 8.2 | 4.1 | 4.2 | 4.5 | 3.0 | 3.9 | 4.0 |
| Newcastle | 11.9 | 5.7 | 5.0 | 3.9 | 3.8 | 7.3 | 4.3 | 4.4 | 4.7 | 3.6 | 5.0 | 4.9 |
| Broken Hill | 10.6 | 6.6 | 5.6 | 3.9 | 5.2 | 10.0 | 6.2 | 6.6 | 6.6 | 4.2 | 5.6 | 6.1 |
| Goulburn | 11.5 | 5.2 | 4.2 | 3.8 | 3.5 | 6.1 | 4.2 | 4.2 | 5.2 | 3.9 | 4.8 | 4.2 |
| Bathurst | 10.6 | 4.4 | 3.9 | 3.2 | 3.3 | 5.9 | 3.8 | 4.0 | 4.1 | 3.3 | 3.8 | 4.1 |
| Melbourne | 10.2 | 6.2 | 5.0 | 4.0 | 3.6 | 8.7 | 4.4 | 5.0 | 5.1 | 3.1 | 4.1 | 4.2 |
| Ballarat | 10.2 | 7.0 | 0.1 | 4.9 | 4.4 | 9.7 | 5.6 | 5.9 | 6.9 | 4.2 | 5.9 | 4.9 |
| Bendigo | 11.3 | 5.6 | 5.2 | 3.6 | 4.2 | 8.0 | 5.0 | 5.3 | 5.3 | 3.5 | 4.7 | 4.8 |
| Geelong | 11.8 | 5.8 | 5.4 | 4.0 | 3.8 | 7.8 | 4.3 | 5.2 | 5.5 | 3.3 | 4.4 | 4.4 |
| Warrnambool | 11.1 | 5.5 | 4.8 | 3.3 | 4.1 | 6.9 | 4.0 | 4.8 | 4.9 | 3.1 | 4.0 | 4.9 |
| Brishane | 12.1 | 4.4 | 3.3 | 3.4 | 3.0 | 6.4 | 3.8 | 3.7 | 4.4 | 2.9 | 3.9 | 4.6 |
| Toowoomba | $1 \begin{array}{ll}1 & 2.4\end{array}$ | 4.9 | 3.1 | 2.1 | 2.5 | 5.8 | 2.9 | 2.9 | 4.4 | 2.5 | 3.9 | 4.5 |
| Rockhampt'n | 12.9 | 5.7 | 4.8 | 3.4 | 2.9 | 5.9 | 4.0 | 3.7 | 5.0 | 3.9 | 4.6 | 5.3 |
| Charters Towers | 13.8 | 5.0 | 3.5 | 3.5 | 4.0 | 6.0 | 4.0 | 4.0 | 4.5 | 3.5 | 4.5 | 5.5 |
| Warwick | 12.5 | 5.0 | 4.0 | 4.0 | 4.0 | 5.0 | 4.0 | 4.0 | 5.0 | 4.0 | 4.4 | 5.0 |
| Adelaide . . | 11.7 | 5.8 | 4.8 | 4.2 | 3.7 | 8.1 | 4.5 | 4.9 | 5.7 | 3.6 | 4.5 | 4.4 |
| Kadina Moonta \& |  |  |  |  |  |  |  |  |  |  |  |  |
| Wallaroo . | 11.4 | 5.6 | 5.4 | 3.9 | 4.5 | 7.0 | 5.4 | 5.4 | 5.6 | 4.2 | 5.0 | 4.6 |
| Port Pirie | 10.7 | 5.9 | 4.9 | 3.1 | 4.4 | 7.9 | 5.4 | 5.6 | 6.0 | 4.0 | 4.9 | 4.9 |
| Mt. Gambier | 11.8 | 5.2 | 4.6 | 3.6 | 4.0 | 5.9 | 4.0 | 4.0 | 5.2 | 4.0 | 5.0 | 4.1 |
| Petersburg | 1.00 .9 | 5.3 | 5.0 | 4.8 | 4.7 | 7.1 | 5.0 | 5.0 | 5.5 | 4.1 | 4.9 | 4.9 |
| Perth and Fremantle | $1 \begin{array}{ll}1 & 1.4\end{array}$ | 7.7 | 6.6 | 6.1 | 5.9 | 11.8 | 7.2 | 7.2 | 7.0 | 4.6 | 6.0 | 7.9 |
| Kalgoorlie \& Boulder | 14.6 |  |  | 68 | 8.7 | 11.9 | 8.6 | 8.7 | 8.7 | 6.3 | 8.1 |  |
| Mid. Junction | $1 \begin{array}{ll}1 & 4.6\end{array}$ | 8.9 | 7.8 | 6.8 | 8.7 | 11.9 | 8.6 | 8.7 | 8.7 | 6.3 | 8.1 | 8.9 |
| \& Cuildford | $1 \begin{array}{ll}1 & 1.4\end{array}$ | 8.3 | 7.5 | 4.9 | 7.0 | $1 \begin{array}{ll}1 & 0.5\end{array}$ | 8.0 | 7.8 | 7.6 | 5.4 | 6.6 | 8.3 |
| Bunbury . . | $\begin{array}{ll}1 & 1.0 \\ 1 & 2.0\end{array}$ | 9.0 | 8.0 | 5.7 | 7.2 | $1 \quad 0.0$ | 8.1 | 8.1 | 8.3 | 6.1 | 7.3 | 9.0 |
| Geraldton | 12.5 | 7.6 | 6.6 | 4.9 | 6.3 | 0.1 | 6.9 | 7.2 | 7.2 | 5.7 | 6.4 | 7.6 |
| Hobart | 110.8 | 6.4 | 5.4 | 3.9 | 4.5 | 8.4 | 5.3 | 6.0 | 5.9 | 3.5 | 4.4 | 5.3 |
| Launceston | 11.8 | 6.1 | 5.5 | 3.8 | 4.8 | 7.0 | 5.2 | 5.7 | 5.7 | 3.8 | 5.2 | 5.2 |
| Zeehan | 11.4 | 6.8 | 6.6 | 5.4 | 5.9 | 8.0 | 6.4 | 8.8 | 6.8 | 5.8 | 6.5 | 6.9 |
| Beaconsfield | $1 \begin{array}{rr}1 & 0.0\end{array}$ | 6.2 | 5.6 | 4.4 | 5.5 | 6.9 | 6.1 | 6.1 | 5.4 | 4.3 | 5.3 | 6.4 |
| Queenstown | 11.9 | 7.0 | 6.5 | 4.5 | 5.7 | 8.2 | 6.5 | 6.9 | 6.8 | 4.7 | 5.7 | 6.6 |
| Weighted Average | 10.1 | 6.0 | 4.8 | 4.1 | 3.8 | 8.4 | 4.6 | 4.9 | 5.2 | 3.4 | 4.4 | 4.8 |

[^1]Current Retail Prices in Metropolitan and Country Towns, 1912.*—contd.

| Town. | Mutt'n sh'lder per lb. | Mutt'n Loin per lb. | Mutt'n Neck per lb. | Chops Loin per lb. | Chops Leg per lb. | Chops Neck <br> per lb. | Pork Leg per lb. | Pork Loin per lb. | Pork Belly per lb. | Pork Chops <br> per lb. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sydney | d. 3.4 | d. | d. 3.6 | d. 5.4 | d. 4.7 | d. | 7.8 | 8. | d. 7.6 | $d$. 8.7 |
| Newcastle | 4.1 | 4.7 | 4.0 | 4.9 | 5.1 | 4.3 | 6.4 | 7.7 | 6.3 | 7.8 |
| Broken Hill | 5.1 | 5.5 | 4.4 | 6.3 | 6.3 | 5.8 | 9.1 | 9.1 | 8.3 | 9.9 |
| Goulburn . . | 3.6 | 4.2 | 2.9 | 4.2 | 4.2 | 3.0 | 5.8 | 6.0 | 5.6 | 8.0 |
| Bathurst . | 3.5 | 4.0 | 2.8 | 4.1 | 4.2 | 3.6 | 5.8 | 8.1 | 5.9 | 6.2 |
| Melbourne | 3.4 | 4.2 | 3.0 | 5.2 | 4.7 | 3.5 | 6.4 | 7.1 | 6.9 | 7.3 |
| Ballarat . | 4.0 | 5.1 | 3.5 | 5.5 | 5.9 | 4.3 | 7.3 | 8.1 | 8.3 | 8.2 |
| Bendigo . | 3.5 | 4.7 | 3.3 | 5.3 | 5.3 | 4.2 | 6.3 | 6.6 | 6.6 | 7.0 |
| Geelong . | 3.6 | 4.5 | 3.5 | 4.8 | 4.8 | 3.8 | 6.6 | 7.2 | 7.1 | 7.4 |
| Warrnambool | 4.0 | 4.9 | 3.6 | 5.1 | 5.0 | 4.0 | 6.0 | 6.3 | 6.1 | 6.5 |
| Brisbane | 3.1 | 4.8 | 4.3 | 5.1 | 5.1 | 5.1 | 7.0 | 7.2 | 6.0 | 7.9 |
| Toowoomba | 2.8 | 4.5 | $3 \cdot 7$ | 4.9 | 4.9 | 4.9 | 6.1 | 6.1 | 5.8 | 6.3 |
| Rockhampt'n Charters | 3.9 | 5.3 | 4.0 | 5.4 | 5.3 | 5.2 | 7.3 | 7.3 | 6.4 | 7.4 |
| Towers | 4.0 | 5.6 | 3.9 | 6.0 | 6.0 | 5.3 | 8.0 | 8.0 | 6.9 | 8.0 |
| Warwick | 4.1 | 5.0 | 4.7 | 5.0 | 5.0 | 4.9 | 7.0 | 7.0 | 7.0 | 7.0 |
| Adelaide . | 3.6 | 4.5 | 3.4 | 5.1 | 5.3 | 4.3 | 7.1 | 7.9 | 7.8 | 8.0 |
| Kadina <br>  <br> Wallaroo | 3.5 | 4.2 | 3.3 | 4.5 | 5.4 | 4.3 | 6.5 | 6.5 | 6.1 | 6.8 |
| Port Pirie | 4.1 | 4.5 | 4.2 | 5.7 | 5.9 | 5.3 | 6.2 | 6.1 | 6.1 | 6.4 |
| Mt. Gambier | 4.0 | 4.6 | 3.5 | 4.9 | 5.0 | 4.4 | 6.0 | 6.1 | 6.1 | 0.1 |
| Petersburg | 4.1 | 4.7 | 3.7 | 5.4 | 5.8 | 4.7 | 6.0 | 6.0 | 5.8 | 6.0 |
| Perth and Fremantle | 6.7 | 7.4 | 5.9 | 8.0 | 8.1 | 6.7 | 8.9 | 9.0 | 8.1 | 9.6 |
| Kalgoorlie \& Boulder | 7.3 | 8.4 | 7.0 | 8.7 | 9.0 | 8.3 | 10.7 | 10.5 | 9.2 | 11.6 |
| Mid. Junction \& Guild ford | 7.3 | 7.5 | 5.9 | 8.4 | 8.4 | 7.0 | 8.7 | 8.7 | 8.0 | 8.9 |
| Bunbury .. | 8.0 | 8.7 | 7.1 | 9.0 | 9.0 | 7.8 | 9.0 | 9.0 | 9.0 | 9.4 |
| Geraldton | 6.6 | 6.8 | 5.8 | 7.6 | 7.6 | 6.9 | 8.0 | 8.0 | 6.5 | 8.0 |
| Hobart | 4.4 | 5.1 | 4.0 | 6.1 | 6.0 | 4.8 | 6.2 | 6.5 | 6.5 | 7.0 |
| Launceston | 4.8 | 5.1 | 3.9 | 6.0 | 5.9 | 5.0 | 6.1 | 6.3 | 6.3 | 6.5 |
| Zeehan | 6.0 | 6.8 | 5.7 | 6.9 | 7.0 | 6.1 | 7.5 | 7.5 | 7.1 | 7.8 |
| Beaconsfield | 5.1 | 5.3 | 4.2 | 6.2 | 6.1 | 5.9 | 6.2 | 6.2 | 6.2 | 6.5 |
| - Queenstown | 6.3 | 6.3 | 5.6 | 6.9 | 7.0 | 6.3 | 7.0 | 7.0 | 7.0 | 7.1 |
| Weighted Average .. | 3.8 | 4.7 | 3.7 | 5.5 | 5.2 | 4.3 | 7.2 | 7.7 | 7.2 | 8.0 |

* Average prices for first 9 months only.

APPENDIX IV.

Weekly House Rents $\dagger$ in Metropolitan Towns, 1901 to 1912.*


[^2]
## APPENDIX V.

Current Weekly House Rents $\dagger$ in Metropolitan and Country Towns, 1912.*

| Tows. | Average Predominant Weekly Rnnts for Hovses having- |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Under <br> 4 Rooms. | 4 Rooms. | 5 Rooms. | 6 Rooms. | 7 Rooms. | Over <br> 7 Rooms. | Weighted Average for all Houses. |
|  | s. d. | s. d. | 8. $\quad$ d. | s. d. | 8. d. | s. d. | 8. d. |
| Sydney . . | 115 | 150 | 1710 | 216 | 260 | 319 | 195 |
| Newcastle | 58 | 75 | 1010 | 138 | 175 | 238 | 108 |
| Broken Hill .. | 6 6 | 97 | 123 | 1310 | 1711 | 23 3 | 104 |
| Goulburn . . | 59 | 74 | 126 | 159 | 221 | $29 \quad 7$ | 143 |
| Bathurst | 57 | 76 | 95 | 120 | 161 | 218 | 106 |
| Melbourne | $8 \quad 9$ | $11 \quad 7$ | 145 | 180 | 215 | $25 \quad 2$ | 158 |
| Ballarat | 46 | 510 | 82 | $10 \quad 9$ | 136 | $20 \quad 5$ | 102 |
| Bendigo | $4 \quad 10$ | 70 | 92 | 11.9 | 1410 | 224 | 103 |
| Geelong | 411 | 88 | 11.6 | $15 \quad 2$ | 208 | $24 \quad 4$ | 137 |
| Warrnambool | $5 \quad 4$ | 8 6 | $10 \quad 10$ | 13 3 | 1511 | 206 | 120 |
| Brisbane | 61 | 81 | $10 \quad 4$ | $13 \quad 9$ | $16 \quad 11$ | 30 | 127 |
| Toowoomba . | 50 | 75 | 105 | 125 | 149 | 221 | 128 |
| Rockhampton . . | $5 \quad 6$ | 75 | 90 | 119 | 1311 | $16 \quad 4$ | 1010 |
| Charters Towers | $4 \quad 10$ | $7 \quad 3$ | $10 \quad 1$ | 1110 | 158 | $20 \quad 11$ | 95 |
| Warwick . | 62 | 811 | 1011 | 13 3 | 168 | 201 | 1211 |
| Adelaide | 911 | 141 | 18 8 | 22. | 259 | 298 | 18 3 |
| Moonta, \&c. . | 58 | 70 | 86 | 112 | 140 | $14 \quad 7$ | 811 |
| Port Piric | 711 | 100 | 11.9 | 138 | 1510 | 1810 | 114 |
| Mt. Gambier . . | 56 | 72 | 94 | 11.6 | 143 | 175 | $10 \quad 1$ |
| Petersburg .. | 80 | 100 | 126 | 150 | 176 | 200 | 127 |
| Perth .- | - 88 | 115 | $14 \quad 3$ | $17 \quad 2$ | 2011 | $27 \quad 0$ | 137 |
| Kalgoorlie .. | 98 | 139 | 17 1 | 206 | $23 \quad 9$ | $32 \quad 2$ | 134 |
| Mid. Junction | 6 | 811 | 128 | $15 \quad 9$ | 187 | $24 \quad 2$ | 117 |
| Bunbury | 84 | 105 | 12.5 | $15 \quad 3$ | 186 | 215 | $10 \quad 6$ |
| Geraldton .. | 117 | 165 | 204 | 228 | $25 \quad 5$ | 342 | 1610 |
| Hobart .. .. | 73 | 98 | 1110 | 143 | 176 | 208 | 1210 |
| Launceston | 66 | 92 | 1111 | 147 | 169 | 21 0 | 129 |
| Zeehan .. | 46 | 610 | 102 | 126 | 140 | 176 | 71 |
| Beaconsfield | 30 | $4 \quad 4$ | $4 \quad 10$ | 59 | 69 | 76 | 48 |
| Queenstown .. | 68 . |  | 105 | 138. | 164 | 200 | 92 |
| Weighted Average | 72 | 95 | 120 | 149 | 1.84 | $23 \quad 7$ | 124 |

* First 9 months only. + The rents are shewn to the nearest penny.


## APPENDIX VI.

Average Annual Wolesale Prices in Melbourne, 1871 to 1912.

| Commodity. | UNIT | 1871. | 1872. | 1873. | 187 | 1875. | 1876. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group I. Meta |  |  |  |  |  |  |  |
| Iron- Pig Mixed Nos. | ton | ${ }^{99}$ 9\% | 1550 | 17611 | 1638 | 124. | ${ }^{\text {s. }} 950$ |
| lod and Bar |  | $207{ }^{0}$ | 2897 | 331 | $320{ }^{9}$ | 263 2t | 223 |
| Angle and Tee |  | 2270 | 2957 | 360 | 33611 | 27931 | 236 |
| Plate .. |  | 2418 | 37310 | ${ }_{301}^{401}$ | 35611 | 279 | 249 |
| Hoop ${ }_{\text {Galvanised }}$ corrugated | " | $\begin{array}{ll}230 \\ 562 & 6 \\ 50\end{array}$ | $\begin{array}{ll}331 \\ 729 & 61 \\ 7\end{array}$ | $\begin{array}{ll}334 & 7 \frac{7}{2} \\ 724 & \\ \\ 48\end{array}$ | ${ }_{691} 33$ | -301 5 | ${ }_{584}{ }^{5} 8$ |
| Gavanised Corrugated | " | 3450 | 44810 | 4900 | $\begin{array}{rrr}691 & 9 \\ 516 & 11\end{array}$ | 657 420 48 | [584 ${ }_{358}{ }^{\text {91 }}$ |
| Zinc-Sheet $\quad$. | " | 5270 | 65111 | 750 | 74810 | 677 | 614 |
| Lead--Sheet | ", | $526 \quad 3$ | 4893 | 580 | 659 | 810 81 | 564 |
| Piping |  | 5200 | 520 0 | 586 | 58474 | 560 0 | 560 |
| Copper-Sheet | db. | 10 | $11 \pm$ |  | 1 1f | 12 | 1 |
| Coal (on Wharf) ${ }^{\text {cos }}$ - | ton | *20 | *28 5 | 39 84 | 3371 |  | $30 \quad 6 \mathrm{t}$ |
| Group II.Textiles, Lbather, \&c |  |  |  |  |  |  |  |
| Jute Goods- Branbags $\begin{array}{r}\text { Cornsacks } \\ \\ \ldots\end{array}$ | dozen | ${ }^{7} 1898$ | $\begin{array}{rr}8 \\ 8 & 7 \\ 13\end{array}$ | $\begin{array}{rr}8 & 39 \\ 13 & 24 \\ & 24\end{array}$ | 7 3 <br> 11 23 <br>   | 5  <br> 9 61 <br> 9 5 <br> 8  | $\begin{array}{ll}5 & 51 \\ 7 & 81\end{array}$ |
| Woolpacks | en'ch |  | 488 | 4 | 1  <br> 3 64 | 308 | 264 |
| Leather-Kip .. | 1b. | 1117 | $2{ }_{3}{ }^{2 t}$ | $2{ }_{3} 2$ | ${ }_{3}^{2} 0 \frac{1}{2}$ | ${ }_{2}^{2} 1$ | 191 |
|  |  |  | 310 | 3 7\% |  |  | 0 |
| Basils | dozen | 145 | 158 | 150 | 130 | 13 4 ${ }^{\text {d }}$ | 130 |
| Cotton-Raw | lb. | 0 7 | 0 91 | 088 |  |  |  |
| Silk-Raw | " | 217 7 | 2151 | 20117 | 1097 | 154 | 192 |
| Wool | " |  | 13 | 131 | $12{ }^{\text {a }}$ | $14 \frac{18}{8}$ | 13 f |
|  | bushel | 4 ${ }^{\text {E }}$ | 6 3 | $511 \pm$ |  |  | 98 |
| Flour | ton | 2986 | 2960 | 2730 | 2712 | 2309 | 254 |
| Bran | bushel | $1{ }^{1} 1$ | 1 11 | 13 H | 16 | $15^{\frac{1}{2}}$ | 1 |
| Pollard |  | $1{ }^{1} 2$ | $1{ }^{1} 1$ | 123 | $17 \pm$ | 15 \% | 4 |
| Oats |  | 358 | 31 | 4 31 | 5 5 ${ }^{\frac{1}{8}}$ | 3118 | 0 강 |
| Ontmeal | ton | 420 | 4074 | 526 | 640 | 5443 | 4093 |
| Barley-Malting | bushel | $\begin{array}{ll}4 & 3 \\ 3\end{array}$ | 581 | - $63 \pm$ | $511 \pm$ | 569 |  |
| Mate Feed |  |  | 310 | $4{ }^{4} 4$ |  | 3102 | 31 |
| ${ }_{\text {May }}^{\text {May }}$ | ton |  | $\begin{array}{rr}3 & 48 \\ 105 & 5\end{array}$ | ${ }_{13} 10{ }^{1}$ | $\begin{array}{rr}5 & 31 \\ 120\end{array}$ | 5  <br> 118 27 <br> 8  | $\begin{array}{r}4 \\ 4 \\ 14 \\ \hline 8\end{array}$ |
| Striay ${ }^{\text {Hay }}$ | ton | $\begin{array}{rr}103 & 4 \\ 55 & 0\end{array}$ | 105 53 50 9 | $\begin{array}{rr}133 & 4 \\ 75 & 5\end{array}$ | $\begin{array}{rr}120 & 0 \\ 75 & 5\end{array}$ | $\begin{array}{rr}118 & \mathbf{9} \\ 75 & \mathbf{3}\end{array}$ | $\begin{array}{rrr}143 \\ 75 & \mathbf{9}\end{array}$ |
| Peas . . | bushel | 4 2t | $\begin{array}{ll}3 & 47\end{array}$ | 4 23 | $5 \mathrm{6t}$ | 47 | $3{ }^{7}$ |
| Potatoes | ton | $49 \quad 9$ | 70 21 | 60 112 | $95 \quad 7 \frac{1}{2}$ | 98 112 |  |
| Group IV. Datry Prodtee- Hami.. a | lb. |  | 010 | 0 104 |  |  |  |
| Bacon . |  |  | 088 | 088 | 08 | 091 |  |
| Cheese | ", | 0 63 | 077 | 081 | 0 91 | 09 |  |
| Butter | " |  | $010 \frac{3}{3}$ | $010 \frac{7}{7}$ | 11 | 138 | 142 |
| Lard .. |  |  |  | ${ }^{0} 68$ | $0{ }^{0} 78$ | ${ }_{0} 8$ | $\begin{array}{ll}0 & 8 \\ 1\end{array}$ |
| Wggs .. | dozen | $\begin{array}{ll}1 & 2 \frac{3}{3} \\ 0\end{array}$ | $\begin{array}{ll}1 & 23 \\ 0\end{array}$ | $1{ }^{1} 27$ | ${ }^{1} 838$ | 1. $4 \frac{1}{8}$ | 43 |
| Honey ${ }_{\text {Grour }}$ Gröceriss, |  |  |  | 0 37 |  | 0 4t |  |
| Currauts | lb. |  |  |  |  |  | 5 |
| Raisins |  |  |  |  |  |  |  |
| Herrings .. | $\left\|\begin{array}{c} \text { doz. } 1 \mathrm{lb} \\ \text { tins } \end{array}\right\|$ |  |  |  |  |  |  |
| Salmon |  | 11 11 ${ }^{\frac{1}{4}}$ | $12 \quad 74$ | 15 37 | 12 8 ${ }^{\text {d }}$ | 96 |  |
| Sardiues | doz. hlvs | 125 | $\begin{array}{ll}13 & 48 \\ 7\end{array}$ | 115 |  | 8114 |  |
| Ten - | Ib. |  |  | 174 |  | 173 | 178 |
| Coffee | , | ${ }^{0} 1118$ | $1{ }^{1} 0$ |  | $1{ }_{1}^{1} 57$ | 14.4 | 15 |
| Cocoa |  | ${ }_{85}{ }^{1} 2^{27} 8$ | $888{ }^{1} \quad 314$ | ${ }_{84}^{1} \quad 31$ | $1{ }^{1} 4$ |  | $1{ }^{37}$ |
| Sugar | ton | 8520 | $882{ }_{0}^{1}$ | 843 | 7631 | 7320 |  |
| ( Mncaroni | lb. |  | ${ }_{0}^{0} 10$ |  | 0 10 <br> 0 28 <br> 1  | 0 98 <br> 0 28 <br> 1  | $\begin{array}{ll}0 \\ 0 \\ 0 & 9 \\ 21 \\ \end{array}$ |
| Rice | ton | 4310 | $402{ }^{4}$ | 466 | $489{ }^{3}$ | 446 | $382{ }^{6}$ |
| Salt-Fine $\because$ |  | 848 | 90 | 929 | 9711 |  | 710 |
| Rock.. |  | 74 | 1160 | 943 | 924 | 1098 |  |
| Mustari | $\underset{\text { dins }}{ }$ |  |  |  |  |  |  |
| Starch | lb. | 05 | 053 | 0 5 5 | $0{ }^{0} 6{ }^{2}$ | $0{ }^{1} 6$ | $0{ }^{5}$ |
| Bluc |  |  | $010 \pm$ | 011 | 011 | 011 | 011 |
| Matches | gross | 50 | ${ }_{4}^{4}{ }^{3}$ | $4{ }^{4} 17$ |  | 4 0t | $4{ }^{4} 0$ |
| Candles | lb. |  | 0 11t | 0 11 ${ }^{\frac{1}{6}}$ | 0104 | $010 \pm$ | $0{ }^{10} 8$ |
| Kerosene ${ }_{\text {Kobec }}$ | gallon |  |  | ${ }_{2}^{2} 3$ | 1 11发 | $1{ }^{1} 9$ |  |
| Tobaceo Grote VI Meat - | lb. |  | 3 34 |  |  | 378 |  |
| Beef .. .. | 100 lb . |  |  |  |  |  |  |
| Mutton | lb. |  |  |  |  |  |  |
| Lamb | each |  |  |  |  |  |  |
| Yeal .. | 1 b . |  |  |  |  |  |  |
| Pork | " |  |  |  |  | $\cdots$ | . |
| Timber--Flooring-6. $\times 1$ | 100ft. lin |  | 1031 |  |  |  |  |
| ${ }_{6}^{6 \times x}$ |  | 8 4 4 | 94 | 115 | 12 1t | 1011 | 1034 |
| $6 \times$ | " | $7{ }^{7} 4$ | ${ }^{6} 114$ | ${ }_{9}^{9} 3$ | 1011 ¢ | $8{ }^{8} 4$ | $8{ }^{89}$ |
| Weatherboards ${ }^{6 \times 8}$ |  | 5 5 5 | 6 58 <br> 6 98 <br> 8  | 6 91 <br> 6 81 <br>   | 880 | ${ }_{6}^{6101}$ | 7 <br> 8 <br> 1 |
| Oregon .. | 1000 ft . sp | 1840 | 1728 | 29610 | 1938 | 1527 | 1558 |
| Cement Shelving .. |  | 2364 | 274 91 | 2893 | 210104 | 2193 | 227 21 |
| Cement ${ }^{\text {White }}$ Lead | cask | 165 | 1611 | 218 | 2110 | 17 31 | 162 |
| White Lead ${ }^{\text {L }}$ - | ton | 8000 | 800 | 8486 | 8524 | 8172 | 840 0 |
| Group Viti. ChemionlsCream of Tartar |  |  |  | 13 3 | 13 |  |  |
| Carbonate of Soda ${ }^{\text {a }}$ | ton | $309{ }^{1}$ | 37611 | $476{ }^{6}$ | 495 | $392{ }^{1}$ | $3331{ }^{1}{ }^{36}$ |
| Saltpetre |  | 7540 | 872 | 817 | 730 | 6943 | ${ }_{606} 12$ |
| Sulphur | " | 3280 | 333 | 2824 | 302.4 | 2654 | 295 |

Average Annual Wholesale Prices in Melbourne, 1871 to 1912-contd.


Average Annual Wholesale Prices in Melbourne, 1871 to 1912-contd.


Average Annual Wholesale Prices in Melbourne, 1871 to 1912-contd.


Average Annual Wholesale Prices in Melbourne, 1871 to 1912-contd.


Average Annual Wholesale Prices in Melbourne, 1871 to 1912 -contd.


Average Annual Wholesale Prices in Melbourne, 1871 to 1912-contd.

Commodity


## $E$.

APPENDIX VII.
FORMS USED FOR COLLEOTION OF DATA AS TO RETAIL PRICES AND HOUSE RENT
Town $\qquad$ State $\qquad$
Ofice No. R.P. 9/...
RETAIL PRICES.

Name

This hall-sboet is to bo detrehed and postod to the "Commonwenth, Statistici inn." Molbourne.

In filing in this form pleast follow Instructions carefully. dancrary, 191.


- seo intituctions, paragriph is

Remankis (Enter bere the cause of may matorial adranco or decline is the prite of any article sineo the middle of lath month)..............

(Continue remarts on back of sheot, it deceseary)


District or Suhurh_—_Ofice No. R.P. 161... Office No. R.P. 1
WEEKLY HOUSE RENTS.
Particulars requirad under the Consus and Statistics, Act, $1800^{\circ}$.


## APPENDIX VIII.

# THEORY OF DETERMINING PRICE-TNDEXES SHEWTNG VARIATIONS IN THE EXCHANGE-VALUE OF GOLD, OR IN THE COST OF LIVING. 

BY G. H. KNIBBS, C.M.G., F.R.A.S., F.S.S., Etc., Etc.<br>Federal Statistician, Australia.

## SYNOPSIS.

1. General theory of determining price-indexes.
2. Price-indexes from relative total expenditures and from price-ratios.
3. Arithmetic, geometric, and harmonic means.
4. The harmonic mean is really as legitimate as the arithmetic, but is not more so.
5. Weights to be applied when price-ratios are used.
6. Computations of mean weights.
7. Error of means.
8. Index-numbers referred to average conditions during a period.
9. Differences between various price-indexes.
10. Various methods adopted for measuring the exchange-value of money.
11. Supposed defects in the geometric mean.
12. The aggregate expenditure method the best.
13. Conclusion.
14. General Theory of Determining Price-Indexes.-The exchange-value or purchasing-efficiency of money is measured by the amount of any commodity which a unit of money (fl. say) will purchase; or it is measured in a reciprocal way by the amount of money or price which has to be paid for a unit of the commodity in question. The latter measure is, of course, a reciprocal of the former, that is, the exchange-value or money-purchasing efficiency of the commodity is measured by the quantity of money which a unit of the commodity will buy, or for which it can be exchanged. It is convenient, and is the custom, to express exchange-values through price. When the price of a commodity changes (for example, when it becomes greater) it denotes change of (reduced) efficiency in the purchasing-power or exchange-value of money with respect to that commodity. Thus if a thing that originally cost $£ 4$, at some later date costs $£ 5$, the price has advanced in the ratio from 1 to 5 or $25 \%$, or the efficiency of the purchasing-power or exchange-value of money has, in respect of the commodity in question, fallen from 1 to $\frac{4}{b}$, or $20 \%$, the two statements being virtually the same. The ratio of the price at one date to that at another is called its price-ratio in respect of those dates. It has become customary for economists to regard every instance of a rise or fall in price in a particular commodity as an individual measure of a variation in the exchange-value of money, a measure which has value or weight in proportion as expenditure upon the commodity in question enters into the aggregate of expenditure upon the whole series of commodities of which it forms a single member. The term "exchange-value" is to be preferred because it is' unambiguous; "value" without qualification might denote utility-value, esteem-value, cost-value, etc. Here it may be remarked that the method of determining variations of the exchange-value of gold by means of priceratios, is not a good one, as is shewn hereinafter, and the only satisfactory method is that of aggregate expenditures for a given regimen.

Now it is obvious that if, in a series of commodities, the quantity used in a given period be constant for each commodity, the measure of the economic importance or economic " weight" of each is the relative expenditure in money units on that member of the series.* Hence through statistics we may obtain some idea of this measure or "weight." Weight in this sense has, of course, no direct connection with physical weight.

When prices have changed, however, the "weights" will have changed also, unless the quantities of the commodities have changed so as to leave the expenditures (or quantities multiplied by the price) the same. Ordinarily it may of course be said that the " weights" will have changed. Now there can be no real comparison of the relative purchasing-power or exchange-value of money, except on some supposition of constancy in human requirements, and just in proportion as the usage of different commodities varies so will any estimate of relative purchasingpower become dubious. In short, a fixed regimen is essential for an accurate determination.

In some instances human requirements are fairly constant. If we suppose that, for an "average" member of the community, a particular regimen be adhered to, then clearly we may tabulate the aggregate expenditure on that regimen at two dates; and the expenditure thereon at the later date, divided by the expenditure at the former, will measure the expenditure-ratio for the two dates. Thus, for example, if we suppose it to increase, it will represent a rise in the cost of the commodities. The reciprocal of this ratio or relative increase measures the decrease in the purchasing-power of money with respect to the particular regimen.

If the regimen itself vary, any computation of the change in question is dubious, because it contains two elements, viz. :-
(i.) Change in the regimen itself, i.e., change in the use of the commodities (or standard of living), and
(ii.) Change in the expenditure on the cost of the individual elements in the regimen.
Where the regimen changes either in. virtue of the changes in price, arbitrarily, or in response to changes in the " standard of living," etc., there are still assumptions by means of which accurate comparisons can be made. Thus we may make several definite suppositions, for example :-(a) that the quantities at the former date apply to the later, and thus compute what the effect of changed price would be; or (b) we may, on the other hand, suppose that the quantities used at the second date were actually those at the earlier date, and can again compute the aggregate cost of the regimen on this assumption. Both of these comparisons are, in their way, valid, intelligible, and respond to certain questions of sociological importance that from time to time arise, and which for certain purposes demand an answer. The best general assumption (c) is, of course, that some mean-value of the " weights" applies : this mean may be arithmetic, geometric, or harmonic ; and any one of these means may naturally arise. It is shewn hereinafter that the geometric mean is doubtless the most accurate generally, but that in certain cases the arithmetic may be used.

If we have price-ratios for a series of commodities, and deduce from them some general ratio that expresses for the series in question the price on the whole at the second date, as compared with the former, such a ratio is called the price-index of the latter date.

The nature of the combination of the price-ratios in the calculation of a priceindex, even when the relative weights are decided, is a matter for consideration. It is essential, for example, for satisfactory comparisons that a series of priceindexes which profess to express changes in the purchasing-power of money shall furnish the same relation between the purchasing-power at any two dates, as would be furnished by calculating by the method approved from the original data for the two dates. If this were not so, then obviously the index-numbers do not fulfil their profession ; in short, they are misleading.

Index-values, as ordinarily furnished, are unfortunately subject to this criticism, viz., that they cannot, in the nature of the case, be assumed to represent intelligibly the relation required, at least with sufficient precision to answer many practical questions. This may be readily seen by comparing any two series of price-indexes.
2. Price-Indexes from Relative Total Expenditures and from Price-Ratios.-For a series of commodities $\mathrm{A}, \mathrm{B}, \mathrm{C}$, the price at a certain date is $a_{0}, b_{0}, c_{0}$, etc.; at some later date it is $a_{1}, b_{1}, c_{1}$, etc. The quantities of these commodities may be denoted by $a, \beta, \gamma$, etc., with suffixes 0 and 1 according to the date. The unit by which any commodity $\mathbf{A}, \mathrm{B}$, or C , etc., is measured may of course be anything whatever,

[^3]as a pound avoirdupois, a gallon, a gross, an article, etc. The price-ratio at the latter date as compared with the former ordinarily varies with the commodity used to determine it. Thus it may be written :-
\[

$$
\begin{equation*}
{ }_{0} p_{1}=\frac{a_{1}}{a_{0}} ;{ }_{0} q_{1}=\frac{b_{1}}{b_{0}} ; \quad{ }_{0} r_{1}=\frac{c_{1}}{c_{0}} ; \text { etc. } \tag{l}
\end{equation*}
$$

\]

according as commodity $\mathrm{A}, \mathrm{B}$, or C , etc., is used. In attempting to utilise these price-ratios $p, q, r$, etc., for any general deduction, the relative-weight which should be ascribed to each obviously demands consideration. If the quantities or mass-units used were identical at each date, and were, say, $a, \beta, \gamma$, etc., we should have a general price-ratio $I$, determined as follows, viz. :-

$$
\text { (2) } \ldots \ldots{ }_{0} I_{1}=\frac{P_{1}}{P_{0}}=\frac{a a_{1}+\beta b_{1}+\gamma c_{1}+\text { etc. }}{a a_{0}+\beta b_{0}+\gamma c_{0}+\text { etc. }}
$$

$P_{0}$ and $P_{1}$, denoting total expenditure on commodities $\mathrm{A}, \mathrm{B}, \mathrm{C}$, etc., and $I_{1}$ the deduced general price-index for the dates in question. This formula is one which, for example, would give the relative cost of living at the two dates, on the assumption that the commodities, $A, B, C$, etc., represented the standard of living, and that the quantities of them which were consumed were identical at the two dates. The formula given (2) is unquestionably the only proper formula to use in such a case, and it may be shewn that it is the best formula in all cases. See hereinafter.

To clearly illustrate the matter, suppose, for example, that at the two dates this consumption for some given unit of time was, let us say, uniformly ten 4 -lb. loaves, 1 pound of tea, and 5 quarts of milk.* Suppose further that the prices at date 0 were respectively 5 d . per loaf, 15 d . per lb ., and 6 d . per quart, and at date $1 \mathrm{6d}$. per loaf, 24d. per lb., and 4d. per quart. Then on this assumption the actual cost of living (so far as these items are concerned) would have risen from 95 d . to 104 d ., that is in the ratio of 100 to 109.47 , or in other words, there would be a $9.47 \%$ increase in the " cost of living." $\dagger$

A method very commonly employed, however, for estimating changes of this kind is to ascertain the price-ratio for each commodity, that is, to find the quantities $p, q, r$, etc., by dividing the price per unit, at the second date by that at the first, and to take a mean of all for a general result. $\ddagger$ The price-ratio is, of course, independent of the size of the unit. These quotients are commonly multiplied by 100 for mere convenience.

As reliance upon price-ratios and combinations of them is very common, the question will be referred to at some length.

If price-ratios were really of equal weight we should have

$$
\begin{align*}
& \text { (3) } \ldots \ldots I^{\prime}=\frac{1}{n}(p+q+r+\text { etc. to } n \text { terms }) ; \text { or } \\
& \text { (4) } \ldots \ldots I^{\prime \prime}=\frac{\dot{n}}{\sqrt{\prime}(p . q . r . \text { etc. to } n \text { terms }) ; \text { or }} \\
& \text { (5) } \ldots \ldots I^{\prime \prime \prime}=\frac{1}{\frac{1}{n}\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}+\text { etc. }\right)}=\frac{n(p . q . r . \text { etc. })}{(q r \ldots)+(p r \ldots)+(p q . .)+\text { etc. }} \tag{5}
\end{align*}
$$

according as we preferred the arithmetic, the geometric, or the harmonic mean. Which of these is to be preferred is a point to which we shall refer later. The arithmetic, geometric and harmonic means all assume that each commodity is of equal importance in the result, but which is the proper one to adopt depends on other considerations of a more complex character. Popularly the arithmetic mean, viz., the ordinary average, is supposed to be satisfactory, but this is an error arising ordinarily from the fact that what underlies such an assumption is not apparent. Taking

[^4]the example just quoted, and regarding the evidence of each commodity as to rise of price as of equal value, we should have the following results according as we take one or the other mean, viz. :-


Thus we have four results, viz., by formula (2), viz., the ratio of total expenditure $109.47 \%$; by formula (3) based on the unweighted arithmetic mean 115.56 ; by formula (4) based on the unweighted geometric mean 108.58 ; and by formula (5) based on the unweighted harmonic mean 101.41. And it may be added that had we used formula (2) with one unit of each (i.e., $a=\beta=\gamma=1$ ) we should have obtained the result. $130.77 \%$, or $30.77 \%$ increase, and further that by a method given hereinafter we should obtain 109.53.

The illustration shews conclusively that the weight assigned to each is of great importance, but before dealing with this it is necessary to consider how several means can arise in determining price-indexes by means of price-ratios.

## 3. Significance of Arithmetic, Geometric, and Harmonic Means of Price-Ratios..-

That there may be different means has already been referred to. A word is necessary as to their nature. When an increment to any quantity is uniform and independent of the magnitude of the quantity itself, that is, when it is dependent merely upon the interval of time elapsing, and is equal for equal times, then the progression is arithmetic. This is expressed by the following, viz. :-

$$
\text { (6) } \ldots \ldots(a) ; \quad a+\frac{b-a}{2}=\left(\frac{a+b}{2}\right) ; \frac{a+b}{2}+\frac{b-a}{2}=(b) \text {; }
$$

the common difference being $\frac{1}{2}(b-a)$, and the quantity $\frac{1}{2}(a+b)$ being the arithmetic mean of $a$ and $b$. Often, however, in the nature of the case the magnitude of the increase is actually dependent on the magnitude of the quantity to which it is added; for example, compound interest on money, that is, the rate of increase, is constant : then the progression is geometric, for example :-
(7)

$$
(a) ; a \times \sqrt{\frac{b}{a}}=(\sqrt{a b}) ; \sqrt{\overline{a b}} \times \sqrt{\frac{b}{a}}=(b),
$$

the common ratio being $\sqrt{ }(b / a)$ and $\sqrt{ }(a b)$ denoting the geometric mean of $a$ and $b$. We thus see that the square-root of the product of $a$ and $b$ is the mean value, when $a$ increases to $b$ at a constant rate on the increasing quantity. There is another important way in which a quantity can grow. Suppose $a$, in the fraction $\frac{1}{a}$, increases (or diminishes) to $b$, in the fraction $\frac{1}{b}$, independently of the magnitude of $a$ (or of $b$ ) Then we have :-

$$
\text { (8) } \ldots \ldots\left(\frac{1}{b}\right) ; \frac{1}{b-\frac{b-a}{2}}=\left(\frac{1}{\frac{1}{2}[a+b]}\right) ; \frac{1}{b-2\left(\frac{b-a}{2}\right)}=\left(\frac{1}{a}\right)
$$

Multiply these by $a b$ and we get :-

$$
\text { (9) } \ldots \ldots(a) ; \quad\left(\frac{2 a b}{a+b}\right) ; \text { and }(b)
$$

Then either series of the quantities in the brackets in (8) or (9) are in harmonic progression, formula (8) giving the form in which the progression arises in question of change in price-ratios, formula (9) that which is usually given as the harmonic mean between $a$ and $b$.
4. The Larmonic Mean is really as legitimate as the Arithmetic, but is not more so, and both are invalid.-The question of the legitimacy of employing either the arithmetic or the harmonic or the geometric method of arriving at a price-index may readily be illustrated by means of examples. It may be premised that if, at the beginning of a period of time, the price-index be taken as unity, and at the end of the period it is found by any process to be, say $I$, then, starting at the end of the period with a price-index of unity, and working baok by the same process, one should arrive at $1 / I$ as the price-index at the beginning. In other words, to have any definite meaning the ratio between the two index-numbers should always be maintained if the scheme of calculation be arithmetically valid. With this principle as a crucial test, the question arises which, if any, of the three methods of arriving at priceindexes will satisfy the test. Taking the same example as before, where three commodities, whose starting price is unity, changed in price until they stood respectively at $1.20: 1.60$ : and .6667 or $\frac{8}{5}$, 景 and $\frac{3}{3}$ we have-

$$
\begin{aligned}
& \text { Arithmetic Mean }=\frac{1}{3}\left(\frac{6}{5}+\frac{8}{5}+\frac{2}{3}\right)=1.1556 . \\
& \text { Geometric Mean }=\sqrt[3]{ }\left\{\left\{\frac{6}{5} \cdot \frac{8}{5} \cdot \frac{2}{3}\right\}=1.0858 .\right. \\
& \text { Harmonic Mean }=\frac{3}{\frac{5}{6}+\frac{5}{8}+\frac{3}{2}}=1.0141
\end{aligned}
$$

Consequently the new index-numbers are respectively $1.1556,1.0858$, and 1.0141 . The reciprocals of these numbers are respectively $0.8654,0.9210$, and 0.9861 . Consequently if the process of obtaining the index-numbers be reversed, and we start at the end of the period, assuming that the corresponding index-number is unity, and then work back to the beginning by the three processes, we ought to find that the arithmetic gives 0.8654 as the price-index at the start, the harmonic process ought to give 0.9861 , and the geometric ought to give 0.9210 . We will see now what actually does happen. Our three price-ratios become $\frac{5}{6}$, $\frac{5}{8}$ and $\frac{3}{2}$.

$$
\begin{aligned}
& \text { Arithmetic Mean } \quad \frac{1}{3}\left(\frac{5}{6}+\frac{5}{8}+\frac{3}{2}\right)=0.9861 \text {, instead of } 0.8654 . \\
& \text { Geometric Mean } \quad \sqrt[3]{ }\left\{\frac{5}{6} \cdot \frac{5}{8} \cdot \frac{3}{2}\right\}=0.9210 \text {, as before } 0.9210 . \\
& \text { Harmonic Mean }=\frac{3}{\frac{6}{5}+\frac{8}{5}+\frac{2}{3}}=0.8654, \text { instead of } 0.9861 .
\end{aligned}
$$

We thus see that the arithmetic process gives the result expected from the harmonic process, and the harmonic, the result expected from the arithmetic ; but the geometric process gives the result expected from that process. That is, neither the arithmetic nor the harmonic process is reversible, and this is a grave defect, in fact a fatal one, as regards their practical use. The geometric process alone satisfies the indicated test of consistency.
5. Weights to be applied when Price-ratios are used.-Attention may now be given to the important question of weighting, if price-ratios are used. It is obvious that relative units of quantities used in the same period must be employed with the method of expenditures ; formula (2). Reverting to the original illustration, we may further consider the case of the three commodities whose prices, starting at
unity at date 0 , become respectively $1.20: 1.60$ : and 0.6667 at date 1 . We shall denote the weights by $u, v, w$, etc.; they measure relatively the expenditure on the corresponding commodities. Three courses may be adopted.
(i.) The price-ratios can be weighted according to the respective expenditures at date 0 .*
(ii.) The price-ratios can be weighted according to respective expenditures at date 1.*
(iii) The price-ratios can be weighted according to some mean of the two expenditures. Of these mean-weights, there are only three which it is at present proposed to consider, viz., those already referred to ( $a$ ) the arithmetic, (b) the geometric, and (c) the harmonic.*

These deduced mean-weights (iiia), (iii.b) and (iii.c) can be computed by formula (3), (4), and (5) if we substitute $u, v$, and $w$ for $p, q$, and $r$ therein, and the different weights, computed in the way indicated, are shewn in the following table :-

| Method. | Expenditures as at- | Weights. |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $u$ | $v$ | $w$ |
| (i.) | Date 0 | 50 | 15 | 30 |
| (ii.) | Date 1 | 60 | 24 | 20 |
| (iii.a) | Arithmetic mean, dates 0 and $1 .$. | 55 | 19.5 | 25 |
| (iii.b) | Geometric mean, dates 0 and 1.. | 54.77 | 18.97 | 24.49 |
| (iii.c) | Harmonic mean, dates 0 and $1 .$. | 54.55 | 18.46 | 24 |

The respective index-numbers, computed as by formula (10) hereinafter, viz., that which is most commonly used, are given by the amounts

$$
\frac{(50 \times 1.20)+(15 \times 1.60)+(30 \times 0.6667)}{50+15+30}
$$

and four other similar expressions. Their values multiplied by 100 are :

## Index according to-

(i.) $:=109.47 \dagger$;
(iii.a) $=114.44$;
$($ iii.c) $=114.41$.
(ii.) $=118.97$;
(iii.b) $=114.43$;

The last three results, though worthless, are almost identical, but (iii.b) and (iii.c) would, of course, not be employed with formulæ (3) or (10) hereafter. Given the weights to be adopted, we may now consider the question how the price-index should be computed if price-ratios are used at all. We may remark that a " weighted mean" is the mean that would be obtained by regarding each item as repeated the number of times indicated by the weight number.

Let the weights corresponding to commodities $A, B$, and $C$ (or to the priceratios $p, q$, and $r$ ) be denoted by $u$, $v$, and $w$.

Then instead of formulae (3), (4) and (5), we have, using $J$ (instead of $I$ ) to denote a weighted-mean, the following formulae, according to whether we employ the arithmetic, geometric, or harmonic mean :

$$
\begin{aligned}
& (10) \ldots \ldots J^{\prime}=\overline{(u+v+w+\text { etc. })}(u p+v q+w r+\text { etc. }) \\
& (11) \ldots \ldots J^{\prime \prime}=\left\{p^{\prime \prime} \cdot q^{v} \cdot r^{z v} \cdot \text { etc. }\right\} \frac{1}{u+v+w+\text { etc. }} \\
& (12) \ldots \ldots J^{\prime \prime \prime}=\frac{u+v+w+\text { etc. }}{\frac{u}{p}+\frac{v}{q}+\frac{w}{r}+\text { etc. }}
\end{aligned}
$$

[^5]By taking logarithms of (11) we see that we obtain a result analogous in form to (10) since

$$
(11 a) \therefore \ldots \log . J^{\prime \prime}=\frac{1}{u+v+w}+\text { etc. }(u \log . x+v \log . q+w \log . r+\text { etc. })
$$

We see thus, that using the weights indicated by (iii.b) according to formula (11), we have

$$
\log . J^{\prime \prime}=\frac{1}{98.23} \quad\{54.77 \times 0.07918+18.97 \times 0.20412+24.49 \times \overline{1} .82391\}
$$

thus $J^{\prime \prime}=1.0956$; and $100 J^{\prime \prime}=109.56 . *$
Lastly, using the weights indicated (iii.c) according to formula (12) we get

$$
J^{\prime \prime \prime}=\frac{54.55+18.46+24}{\frac{54.55}{1.2}+\frac{18.46}{1.6}+\frac{24}{0.6667}}=\frac{97.01}{92.99}=1.0432 ; \text { and } 100 J^{\prime \prime \prime}=104.32 .
$$

From the examples given it will be noticed that when the price-index is computed as a geometric mean, it lies between values given by the harmonic and arithmetic means, the arithmetic being the highest. Incidentally, it may be remarked that it is obvious that the weighted geometric mean will be lower than the weighted arithmetic mean, since, with numbers greater than unity and very near unity, the difference between the logarithms of numbers is much less than the differences between the numbers themselves. Thus, as we see at once from formula (11a), the effect of large differences of weighting must necessarily be less when the geometric mean is computed, rather than the arithmetic. That on other grounds the geometric mean is also to be preferred can be seen instantly from the fact that it incidentally gives consistent results in whatever way we work from one date to another, whereas the arithmetic and harmonic means do not give consistent results. By parity of process differences of value may in general be appropriately measured by their relation to the quantity which fluctuates, and this conception of rate-variation necessarily leads to the adoption of geometric means: or to put it in other words, -the moment price-variation is looked at from the standpoint of rate-differences (for example, Id. is $10 \%$ in the case of an urticle the value of which is $10 \mathrm{~d} .$, but only $5 \%$ where the value of the article is 20 d .) we see at once that all measurement of change of value may quite appropriately be in rates, and, consequently, the geometric mode of computing may be regarded as the legitimate one where the original data are supplied in the form of price-ratios. Finally, it may be noted that the weighted geometric inean, the weights being given by (iii.b), in conjunction with formula (11), is 109.56 , by (iii.a) and formula (11) is 109.53 , and by the cost-of-living formula, viz. (2), is $109.47 \dagger$; and further if the original weights 50,15 , and 30 for commodities $A, 13$, and $C$ be taken, and the weighted geometric mean of the price-ratios be calculated, we obtain 104.30 (practically identical with the harmonic result of formula (12) just given). This shews that it is by no means satisfactory to use the original weights, as is usually done in the case of price-indexes, unless there be reason to believe they are sensibly constant throughout.
6. Computation of Mean Weights. - We now reach the discussion of the general problem of which the example just given is a particular case. Suppose at date 0 the prices of a series of commodities are respectively $a_{0}, b_{0}, c_{0}$, etc.; and an amount $a_{0}$ is bought of the first, $\beta_{0}$ of the second, etc. ; the total expenditure on the first amounting to $\zeta_{0}$, on the second to $\eta_{0}$, etc. Suppose further that at date 1 the respective prices are $a_{1}, b_{1}$, etc., the price-ratios $\frac{a_{1}}{a_{0}}$, etc., are denoted by $p, q, r$, etc., and the total expenditures by $\zeta_{0}, \eta_{0}$, etc., at date 0 , and $\zeta_{1}, \eta_{1}$, etc., at date 1 . Then weighting the different quantities by the geometric means of the expenditures according to the geometric formula, we heve, since

$$
\begin{array}{r}
(13) \ldots \ldots \zeta_{0}=a_{0} a_{0} ; \zeta_{1}=a_{1} a_{1} ; \text { etc. } \\
(13 a) \ldots \ldots \eta_{0}=\beta_{0} b_{0} ; \eta_{1}=\beta_{1} b_{1} ; \text { etc., } \\
\text { etc., etc., etc., etc., etc., }
\end{array}
$$

for index-number at date 1 ,

[^6]a number whose logarithm is
$$
(15) \ldots . \log . J^{\prime \prime}=\frac{\sqrt{\zeta_{0} \zeta_{1}} \log . p+\sqrt{\eta_{0} \eta_{1}} \log . q+\text { etc. }}{\sqrt{\zeta_{0} \zeta_{1}}+\sqrt{\eta_{0} \eta_{1}}+\text { etc. }} .
$$

Consequently when the total expenditures at any two periods are at all comparable, we may put with sufficient accuracy
$\qquad$

$$
\begin{equation*}
\zeta=\frac{1}{2}\left(\zeta_{0}+\zeta_{1}\right) ; \text { and } \sqrt{\zeta_{0} \zeta_{1}}=\zeta-\frac{1}{8} \frac{\left(\zeta_{1}-\zeta_{0}\right)^{2}}{\zeta}+\text { etc. } \tag{16}
\end{equation*}
$$

The term - $\frac{1}{8}\left(\zeta_{1}-\zeta_{0}\right)^{2} / r$ is so small as ordinarily to be negligible in nearly'all practical cases, since if the expenditure were double and triple we shall have only the following percentage of error in (16), viz.,

$$
\zeta_{0}: \zeta_{1}=1 ; 2 ; \text { error }=5.72 \% ; \text { if }=1: 3 ; \text { error }=13.40 \%
$$

It is evident that, since in formula (11) this error of weight enters into both the numerator and denominator, its effect must be greatly reduced, and it will lead only to a very small relative error indeed. 'In other words, in (11) we may always take

$$
\begin{equation*}
\ldots \ldots u=\frac{1}{2}\left(u_{0}+u_{1}\right) ; v=\frac{1}{2}\left(v_{0}+v_{1}\right) ; \text { etc. } \tag{17}
\end{equation*}
$$

Thus in the expression for the logarithm of the index-number, viz.,

$$
\left(\sqrt{\zeta_{0} \zeta_{1}} \log . p+\sqrt{\eta_{0} \eta_{1}} \log . q+\text { etc. }\right) /\left(\sqrt{\zeta_{0} \zeta_{1}}+\sqrt{\eta_{0} \eta_{1}}+\text { etc. }\right)
$$

no considerable error will be introduced by using arithmetic instead of geometric means, and the computation will be simpler. In order to illustrate this, we may revert to the former example, and consider two commodities whose price-ratios are, as before, 1.2 and 1.6 at the end of some period as compared with the beginning. Let us further take the extreme case where the expenditure on the first commodity is trebled, and that on the second commodity doubled, since this will severely test the validity of the assumption. Thus $\zeta_{0}=50 ; \zeta_{1}=150 ; \eta_{0}=15 ; \eta_{1}=30$; $\log . p=0.07918 ; \log . q=0.20412$; then the two values for the logarithm of the index-number become :-

For log. of geometric mean :-

$$
\frac{50 \sqrt{3} \log .1 .2+15 \sqrt{2} \log .1 .6}{50 \sqrt{3}+15 \sqrt{2}}=\frac{86.602 \log .1 .2+21.21 \dot{3} \log .1 .6}{107.815}=0.10376
$$

For log of arithmetic mean :-

$$
\frac{100 \log .1 .2+22.5 \cdot \log .1 .6}{122.5}=0.10213
$$

These logarithms correspond to index-numbers, which multiplied by 100 , as is usual, are 126.99 and 126.51 respectively, the two results being therefore sensibly identical.
7. Error of Arithmetic Means.-It is worth while to investigate, on the lines of the last example, the amount of error introduced into the logarithm of the priceindex by taking arithmetic instead of geometric means of expenditure.

Suppose, as before, there are two commodities whose price-ratios at date 1 are $p$ and $q$ as compared with unity at date 0 . Suppose that the expenditures at datel are respectively $k^{2}$ and $l^{2}$ times expenditure att date 0 .

By taking arithmetic means the logarithm of the price-index at date 1 becomes :--

$$
\log . J=\frac{\left(1+k^{2}\right) \log \cdot p+\left(1+l^{2}\right) \log . q}{\left(1+k^{2}\right)+\left(1+l^{2}\right)}
$$

By taking geometric means, the logarithm of the price-index at date 1 becomes :-

$$
\log J^{\prime \prime}=\frac{k \log p+l \log q}{k+l}
$$

If $E$ denote the error introduced by taking arithmetic means,

$$
\begin{align*}
& \ldots \ldots E=\frac{k \log \cdot p+l}{k+l} \log \cdot q \cdot \frac{\left(1+k^{2}\right) \log \cdot p+\left(1+l^{2}\right) \log \cdot q}{\left(1+k^{2}\right)+\left(1+l^{2}\right)}  \tag{18}\\
& \quad=(\log \cdot q-\log \cdot p) \cdot \frac{k-l}{k+l} \cdot \frac{k l-1}{k^{2}+l^{2}+2}
\end{align*}
$$

Now we have the inequality

$$
\left(l^{2}+k^{2}\right)>2 k l ; \text { consequently }\left(k^{2}+l^{2}+2\right)>2 k l+2
$$

and therefore
$(19) \ldots, E<\frac{(\log . q-\log \cdot p)}{2} \cdot \frac{k-l}{k+l} \cdot \frac{k l-1}{k l+1}$

This presupposes, of course, that the initial expenditure on each commodity at date 0 was unity.

If the initial expenditures on each commodity, instead of being unity, were respectively $e$ and $f$, then the expenditures at date 1 would be $e k^{2}$ and $f l^{2}$. In this case we have

$$
\begin{equation*}
E=(\log . q-\log \cdot p) \cdot \frac{e f(k-l)(k l-1)}{(e k+f l)\left(e k^{2}+f l^{2}+e+f\right)} \tag{20}
\end{equation*}
$$

In this case the inequality becomes $\left(e k^{2}+f l^{2}\right)>2 \sqrt{e f} . k l$; and $(e+f)>2 \sqrt{e f}$;

$$
\text { consequently }\left(e k^{2}+f l^{2}+e+f\right)>2 \sqrt{e f}(k l+1)
$$

Also it can be shewn algebraically that if $\left(e k^{2}-f l^{2}\right)$ and $(e-f)$ are of the same sign, as is most frequently the case, then

$$
(e k+f l)>\sqrt{e f}(k+l)
$$

for $\left(e k^{2}-f l^{2}\right)(e-f)>0 ;$ consequently $\left(e^{2} k^{2}+f^{2} l^{2}\right)>e f\left(l^{2}+k^{2}\right)$ and $\quad\left(e^{2} k^{2}+f^{2} l^{2}+2 e f k l\right)>e f\left(l^{2}+l^{2}+2 k l\right) ;$ and therefore

$$
(e k+f l)>\sqrt{e f}(k+l)
$$

From this analysis it is evident on reverting to (20) that

$$
E<\frac{\log \cdot q-\log \cdot p}{2} \cdot \frac{k-l}{k+l} \cdot \frac{k l-1}{k l+1}
$$

as in the former case; see (19).
A superior limit has thus been found for the error in the logarithm of the priceindex. As in practical examples $k$ and $l$ are ordinarily nearly equal, the error is thus very small, since $k-l$ will then nearly vanish. A considerable list, viz., of about 50 commodities shews that the error is by no means inconsiderable even when the number of commodities is large.
8. Index-Numbers referred to Average Conditions during a Period.-We have already shewn that the best weight to be adopted in deducing the price-indexes of any two dates is in proportion to the mean of the expenditures, and that no sensible error is involved in taking the arithmetic mean, if the computation as between the priceratios be made on the principle of the geometric mean. But the comparison of the highest value is clearly that based on the average expenditure of a longer period, since the variations of this are less marked, being free from what may be called " large accidental departures from the mean." Hence it is preferable to employ a quinquennium or decennium as basic period. And since it has been established that, for a period covering two dates, the exact nature of the determination of the mean is the weighting to bo adopted (i.e., whether geometric, arithmetic, etc.) is not of high importance, we may get results of a very high order of accuracy in a simple manner. Thus although a strict adherence to theory demands that the logarithms of the price-ratios should be weighted by the geometric means of the two expenditures, still a result identical for all practical purposes can be obtained by using the arithmetic means, and because of the considerable saving in computation secured by using the arithmetic mean of the weights, it is to be preferred. By similar reasoning, the proposition established can be extended to meet the case of a large number of years, instead of only two, and the conclusion is thus reached that if $\zeta_{0}, \zeta_{1}, \ldots \zeta_{m-1}$, are the expenditures at $m$ observed periods, the general weighting may be found by taking the arithmetic mean ${ }_{m}^{1}\left\{\zeta_{0}+\zeta_{1}+\ldots \zeta_{m-1}\right\}$, instead of the theoretically-more-accurate geometric mean $\left(\zeta_{0} \zeta_{1} \ldots \zeta_{m-1}\right) \stackrel{1}{n}$. This is really equivalent to asserting that the basis of the comparison of the purchasingpower of money may be the arithmetic average of the expenditure on the various commodities throughout the period under examination.
9. Differences between various Price-Indexes.-Price-indexes may be said, in general, to purport to represent the relative amount of money that must on the average be paid for commodities at successive dates, the value paid on the original date being taken as 100. Price-ratios are similar to the index-numbers, or priceindexes, but apply only to individual commodities or limited groups of commodities. Since the purchesing power of gold in regard to a particular commodity is an individual measure of its exchange-value (i.e., of the exchange-relation, between the two) it has been commonly imagined that by taking a sufficient number of commodities a general measure of all changes in the purchasing-power of gold can be
determined. In other words, it is supposed that the price-indexes represent the quantity of gold corresponding to 100 units thereof ( $£$ ) at the initial date, viz., that corresponding to the 100 . An examination of the various tables of price-indexes shews that attempts to measure this general relation are very unsatisfactory. To illustrate this the tables hereunder are given. They furnish the price-indexes established by various authorities by computation from various series of commodities, and it is indicated in the tables on what the estimate is based. It will be seen that there are marked divergences between individual results, so great indeed as to indicate that their value is very limited. For one example one series of indexes represents rises, while for the same period another will represent falls. The fact is this, viz., that price-indexes are definite only for a definite regimen, that is for a series of commodities used in given quantities; and the hope to obtain a general price-index which will represent in its totality the variation in the general exchangevalue of gold is to expect the impossible.

No doubt for each country a series of commodities and system of weights might be taken as representing the average usage of the entire population in regard to these commodities. Other series of commodities and systems of weights would represent the usage of the different classes in the community. Both would differ as between nation and nation; consequently if any international standard is to be developed for the widest system of comparisons, the series should be common to all, and the weights should represent the average usage of the nations included. For international comparisons of classes a similar standard-series and average-weights would be required. This has been dealt with elsewhere by me. It will suffice here to observe that a system rendering general international comparisons possible, and also international comparison of classes, would have to be established by an international practice. This could be reached only by an international commission on the subject.

The following tables give the price-indexes furnished by various authorities. They disclose the fact that they are of little value to determine quantitatively small differences of the purchasing efficiency of money, the fact being that such indexes are not sufficiently well-determined to answer many social-economic questions that are arising, for example, an automatic variation of wage-determinations, which has been suggested in this country (Australia). The tables enable one to obtain an idea also of the range of uncertainty as among the methods adopted by different authorities.

TABLE I.-VARIOUS PRICE INDEXES, 1900-1910. REDUCED TO 1900 VALUES AS BASIS.

| Year. | A. | B. | C. | D. | E. | F. | G. | 230 Com. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 22 Com . | 45 Com . |  | 37 Com . |  |  | 257 Com. | $\begin{gathered} \bar{H} . \\ \text { weight- } \\ \text { ed.* } \end{gathered}$ | $\underset{\text { weight'd }}{\text { I. }}$ |
| 1900 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 1901 | 99.5 | 96.9 | 101.9 | 93.5 | 96.9 | 95.2 | 98.2 | 100.2 | 98.9 |
| 1902 | 91.3 | 96.5 | 101.6 | 92.6 | 95.8 | 90.8 | 102.2 | 103.6 | 100.7 |
| 1903 | 93.3 | 96.9 | 103.2 | 92.6 | 94.2 | 90.7 | 102.8 | 103.7 | 102.1 |
| 1904 | 102.6 | 98.3 | 104.3 | 93.6 | 97.8 | 91.8 | 102.3 | 104.5 | 103.0 |
| 1905 | 99.5 | 97.6 | 103.7 | 96.5 | 98.5 | 91.7 | 104.9 | 107.6 | 105.3 |
| 1906 | 108.7 | 100.5 | 103.2 | 102.7 | 103.3 | 97.0 | 110.9 | 113.5 | 110.9 |
| 1907 | 116.9 | 105.7 | 105.8 | 106.2 | 107.7 | 101.9 | 116.3 | 122.1 | 116.6 |
| 1908 | 107.7 | 102.8 | 108.4 | 97.5 | 103.3 | 97.9 | 111.1 | 118.2 | 111.6 |
| 1909 | 102.6 |  | 108.2 | 99.0 | 105.6 | 94.0 | 114.5 | 119.4 | 112.0 |
| 1910 | 111.8 | - | 109.9 | 103.8 | - | - | 119.1 | - | - |

* Weighted according to table of the British Association for the Advancement of Science, 1887 to 1890
A. The Economist (Old Basis!; Wholesale Prices Index Number, 1st January of each Year; 22 Commodities.
B. Board of Trade. Wholesale Prices in United Kingdom; 45 Commodities.
C. Board of Trade. Retail Prices in London.
D. Sauerbeck. Average Prices in England. 37 Commodities.
E. United Kingdom. From Parliamentary Paper Cd. 4867. Imports.
F. United Kingdom. From Parliamentary Paper Cd. 4867. Exports.
G. United States! Wholesale Prices; 257 Commodities.
H. Canada. Wholesale Prices ; 230 Commodities. Weighted.
I. Canada. Wholesale Prices ; 230 Commodities. Unweighted.

TABLE TI.—VARIOUS PRICE INDEXES, 1871-80. REDUCED TO 187.1 AS BASIS.

| Year. | $\begin{aligned} & \mathrm{M} . \\ & 22 \end{aligned}$ <br> Com. | $\begin{gathered} \mathrm{N} . \\ 45 \\ \text { Com. } \end{gathered}$ | $\begin{gathered} \mathrm{O} . \\ 39 \\ \text { Com. } \end{gathered}$ | $\begin{aligned} & \mathrm{P} \\ & 22 \end{aligned}$ <br> Com. | Q. llit Com. | $\frac{R}{50}$ <br> Com. | S. <br> 223 <br> Com. | $\mathbf{T}$ <br> 223 <br> Com. | $\begin{gathered} \mathrm{U} \\ 223 \end{gathered}$ Com. | $\begin{gathered} \mathrm{V} . \\ 223 \\ \text { Com. } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1871 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 1872 | 109.5 | 107.2 | 109 | 106.6 | 106.8 | 105.4 | 102.1 | 97.9 | 97.3 | 103.5 |
| 1873 | 113.8 | 112.3 | 111 | 106.1 | 108.9 | 110 | 101.1 | 96.1 | 94.8 | 99.3 |
| 1874 | 111.6 | 108.9 | 102 | 97.4 | 107.2 | 104.5 | 97.8 | 96.6 | 95.5 | 97.1 |
| 1875 | 107.3 | 103.4 | 96 | 98.2 | 102.2 | 99.1 | 93.8 | 96.0 | 94.7 | 92.1 |
| 1876 | 104.7 | 101.5 | 95 | 96.2 | 101.0 | 92.7 | 86.9 | 92.6 | 90.1 | 95.3 |
| 1877 | 104.8 | 104.1 | 94 | 98.2 | 100.5 | 93.6 | 81.5 | 87.7 | 83.5 | 84.9 |
| 1878 | 98.6 | 97.5 | 87 | 85.4 | 94.9 | 87.3 | 74.5 | 82.6 | 76.9 | 81.3 |
| 1879 | 85.9 | 93.1 | 83 | 83.1 | 92.2 | 83.6 | 71.0 | 77.4 | 69.8 | - |
| 1880 | 99.5 | 95.3 | 88 | 85.0 | 96.0 | 87.3 | 78.6 | 82.9 | 77.1 | - |

M. The Economist (Old Basis); Wholesale Prices; 22 Commodities.
N. Board of Trade. Wholesale Prices in United Kingdom; 45 Commodities.
O. Sauerbeck. Average Prices in England; 39 Commodities.
P. Palgrave's French Prices; 22 Commodities.
Q. Soetbeer's Hamburg Prices; 114 Commodities.
R. Mulhall. "Ratio of Values"; 50 Commodities.
S. Aldrich Report; 223 Commodities-Commodities Unweighted.
T. " ," ," Commodities Weighted according to Uniform Expenditure.
U. ," ", ", Commodities Weighted according to $68.6 \%$ of Total Expenditure.
V.
",
, ," Gold Index Numbers. All commodities averaged simply.
Reverting to Table I., it is obvious that weighting is not a matter of indifference even with a large number of commodities : see columns $H$ and $I$, years 1900 and 1901, for example. Further, it is ovident that the effect of ignoring weighting may be relatively large: see for example the year 1909 in the same columns, giving 119.4 for the weighted, and 112.0 for the unweighted results. It is clear from the same table (see columns A, B, C, D, for example, year 1904) that the indications of tables as now prepared are of relatively small value for deducing reliable estimates.

A comparison of the results on Table II. leads to the same conclusion, viz., that the divergences between different estimates of a price-index are so great as to indicate that at present they are of very limited value.

It will be appropriate to indicate the nature and defects of various methods of computing a price-index. In this connection it may be remarked that if I, J, K, etc., are price-indexes for any series of dates, then the scheme of computation should be such that the ratios $I / J, I / K, J / K$, shall remain unchanged in whatever order the results are obtained.

Some remarks are added later concerning a supposed demonstration that the geometric mean is unsatisfactory.

## 10. Various Methods adopted for measuring the Exchange-Value of Money.-

 The following are various methods which have been employed for determining the variations in the exchange-value of money. The essential features of each method are given in terms of the notation employed hereinbefore. The notation used is that of § 2 , and the products $a a, \beta b$, etc., therefore measure the money-valueimportance of the different commodities at times shewn by the suffix employed. They are denoted by $\zeta_{0}, \eta_{0}$, etc., $\zeta_{1}, \eta_{1}$, etc., according to the dates. See formulw (13) and (13a).(i.) Dutot's Method.-In this method the prices of commodities are taken at their market quotations, and the mass-units are assumed to be equal. Then if $P_{1}$ and $P_{0}$ are the price-indexes at dates 1 and 0 ,

$$
\begin{equation*}
\frac{P_{1}}{P_{0}}=\frac{a_{1}+b_{1}+\text { etc. }}{a_{0}+b_{0}+\text { etc. }}=\frac{a_{0.0} p_{1}+b_{0.0} q_{1}+\text { etc. }}{a_{0}+b_{0}+\text { etc. }} \tag{21}
\end{equation*}
$$

This method consequently weights the price-ratios with the numbers $a_{0}, b_{0}$, etc., viz., the prices at date 0 . The method is probably now rarely used.

## Appendix.

(ii.) Carli's Method.-This method consists simply in taking the arithmetic mean of the different price-ratios and is expressed algebraically as follows :-

$$
\begin{equation*}
\frac{P_{1}}{P_{0}}=\frac{1}{n}\left(\frac{a_{1}}{a_{0}}+\frac{b_{1}}{b_{0}}+\text { etc. }\right)=\frac{1}{n}\left({ }_{0} p_{1}+{ }_{0} q_{1}+\text { etc. }\right) \tag{22}
\end{equation*}
$$

(iii.), Evelyn's Variation of Carli's Method.-In Evelyn's variation several periods are compared with the first, all the prices of which are taken as 100. Suppose that we have

$$
\frac{P_{1}}{P_{0}}=\frac{1}{n}\left(\frac{a_{1}}{a_{0}}+\frac{b_{1}}{b_{0}}+\text { etc. }\right) ; \quad \frac{P_{2}}{P_{0}}=\frac{1}{n}\left(\frac{a_{2}}{a_{0}}+\frac{b_{2}}{b_{0}}+\text { etc. }\right) ;
$$

then it follows that-

$$
\begin{equation*}
\frac{P_{2}}{P_{1}}=\frac{\frac{a_{2}}{a_{0}}+\frac{b_{2}}{b_{0}}+\text { etc. }}{\frac{a_{1}}{a_{0}}+\frac{b_{1}}{b_{0}}+\text { etc. }}=\frac{\frac{a_{1}}{a_{0}} \cdot \frac{a_{2}}{a_{1}}+\frac{b_{1}}{b_{0}} \cdot \frac{b_{2}}{b_{1}}+\text { etc. }}{\frac{a_{1}}{a_{0}}+\frac{b_{1}}{b_{0}}+\text { etc. }} \tag{23}
\end{equation*}
$$

Consequently the expressions $\frac{a_{2}}{a_{1}}, \frac{b_{2}}{b_{1}}$ instead of being weighted evenly (the essential feature of Carli's method) are weighted according to the numbers $a_{1} / a_{0}$, $b_{1} / b_{0}$, etc., i.e., according to the price-ratios between 1 and 0 . This points to an inconsistency in Carli's method, which is repeated in Young's method, to which reference will now be made.
(iv.) Young's Method.-In this method prices at the first period are taken as unity, and at the second period as $a_{1}^{\prime}, b_{1}^{\prime}$, etc. These last values are weighted according to the relative total-exchange-values of the classes in general use (at some period), and the sum of the products divided by the sum of the weights. Algebraically it is expressed thus:-

$$
\begin{equation*}
\frac{P_{1}}{P_{0}}=\frac{\zeta a_{1}^{\prime}+\eta b_{1}^{\prime}+\text { etc. }}{\zeta+\eta+\text { etc. }} \tag{24}
\end{equation*}
$$

$a^{\prime}{ }_{1}$ denoting the value of $\frac{a_{1}}{a_{0}}$ when $a_{c}$ is taken as unity.
Young's method, however, has the same inconsistency as Carli's, for

$$
\begin{align*}
& \quad \frac{P_{2}}{P_{0}}=\frac{\zeta a_{2}^{\prime}+\eta b_{2}^{\prime}+\text { etc. }}{\zeta+\eta+\text { etc. }} ; \text { consequently } \\
& \cdots \cdots \cdot \frac{P_{2}}{P_{1}}=\frac{\zeta a^{\prime}{ }_{2}+\eta b^{\prime}{ }_{2}+\text { etc. }}{\zeta a_{1}^{\prime}+\eta b_{1}^{\prime}+\text { etc. }=\frac{\zeta a_{1}^{\prime} \cdot \frac{a_{2}}{a_{1}}+\eta b_{1}^{\prime} \cdot \frac{b_{2}}{b_{1}}+\text { etc. }}{\Sigma_{\zeta}^{\prime} a_{1}^{\prime}+\eta b_{1}^{\prime}+\text { etc. }}} \tag{25}
\end{align*}
$$

In other words, the weighting is now $\zeta a_{1}^{\prime}, \eta b_{1}^{\prime}$, instead of $\check{\zeta}$, $\eta$, etc.
(v.) Scrope's Method.--The essential feature of Scrope's method is that the same mass-units are employed at different periods. Algebraically it may be written as follows:-

$$
\begin{equation*}
\ldots \ldots \frac{P_{1}}{\ddot{P}_{0}}=\frac{a a_{1}+\beta b_{1}+\text { etc. }}{\alpha a_{3}+\beta b_{0}+\text { etc. }} \tag{26}
\end{equation*}
$$

that is to say, it is what has been called in $\S 2$ herein, the " cost-of-living " formula (2). This is equivalent to the following :-

$$
(20 \mathrm{~b} a) \ldots \ldots \cdot \frac{P_{1}}{P_{0}}=\frac{a a_{0} \cdot a_{\Gamma}+\beta b_{0} \cdot \frac{b_{1}}{b_{0}}+\text { etc. }}{\operatorname{a} a_{0}+\beta b_{0}+\text { etc. }}
$$

This latter formula shews that the price-ratios are weighted by the multipliers $a a_{0}, \beta b_{0}$, etc., which would represent the original expenditures if a, $\beta$, etc., were the original quantities or mass-units, or the average expenditures if $a, \beta$, etc., are suitably taken. Thus it resembles Young's method in form. We shall shew later that it is really the best form.
(vi.) Laspeyres' and Paasche's Variation of Scrope's Methods and Scrope's
"Emended Variation."-It may be remarked that the question of the exact massquantities to be used has not yet been touched. Three methods are possible :-
(a) By using the mass-quantities of the initial period-

$$
\begin{equation*}
\ldots \ldots \frac{P_{1}}{P_{0}}=\frac{a_{0} a_{1}+\beta_{0} b_{1}+\text { etc. }}{a_{0} a_{0}+\beta_{0} b_{0}+\text { etc. }} \text { (Laspeyres' variation) } \tag{26b}
\end{equation*}
$$

(b) By using mass-quantities of the final period;

$$
\begin{equation*}
\ldots \ldots \frac{P_{1}}{P_{0}}=\frac{a_{1} a_{1}+\beta_{1} b_{1}+\text { etc. }}{a_{1} a_{0}+\beta_{1} b_{0}+\text { etc. }} \text { (Paasche's variation); } \tag{26c}
\end{equation*}
$$

(c) By using some mean between the two. The best known is the geometric mean, viz.,

$$
\begin{equation*}
\frac{P_{1}}{P_{0}}=\frac{\sqrt{a_{0} a_{1}} \cdot a_{1}+\sqrt{ } \beta_{0} \beta_{1} \cdot b_{1}+\text { etc. }}{\sqrt{a_{0} a_{1}}, a_{0}+\sqrt{ } \beta_{0} \beta_{1} \cdot b_{0}+\text { etc. }} \tag{26d}
\end{equation*}
$$

which is known as Scrope's "emended variation," see formulæ (13). (13a) and (14) hereinbofore, where it has already been shewn that the more conveniont arithmetic moan of $\alpha_{0}$ and $a_{1}$, etc., is sufficiently accurate.
(vii.) Drobisch's Method.-This method is the best known example of the methods depending on double-weighting. Drobisch took his prices to be prices of the same aggregated mass-unit, that is a unit consisting of all the commodities in the rolative quantities as used. His method assumes that the average price of an aggregated mass-unit will be as follows, viz. :-

$$
\begin{aligned}
& \frac{a_{0} a_{0}+\beta_{0} b_{0}+\text { etc. }}{a_{0}+\beta_{0}+\text { etc. }} \text {, at the first period; and } \\
& \frac{a_{1} a_{1}+\beta_{1} b_{1}+\text { otc. }}{a_{1}+\beta_{1}+\text { etc. }} \text {, at the second period; }
\end{aligned}
$$

and so on. From this we have directly

$$
\begin{equation*}
\frac{P_{1}}{\ddot{P}_{0}} \frac{\frac{a_{1} a_{1}+\beta_{1} b_{1}+\text { etc. }}{a_{1}+\beta_{1}+\text { etc. }}}{\frac{a_{0} a_{0}+\beta_{0} b_{0}+\text { etc. }}{a_{0}+\beta_{0}+\text { etc. }}} \tag{27}
\end{equation*}
$$

This equation is equal to-

$$
\begin{equation*}
\frac{P_{1}}{P_{0}} \frac{a_{1} a_{1}+\beta_{1} b_{1}+\text { etc. }}{a_{0}} a_{0}+\beta_{0} b_{0}+\frac{a_{0}+\beta_{0}+\text { etc. }}{a_{1}+\beta_{1}+\text { etc. }} \tag{27a}
\end{equation*}
$$

lit is oljviously a fallacy to suppose that differently constituted "aggregated-mass-units" can be compared in this way : see remarks in the next sub-section.
(viii.) Lehr's Method.-Lehr's method, as Drobisch's, also employs doubleweighting, but differs from Drobisch in the second factor on the right hand side of (27a). Algebraically it may be written-

$$
\begin{equation*}
\frac{\boldsymbol{P}_{1}}{\bar{P}_{0}}=\frac{a_{1} a_{1}+1 \beta b_{1}+\text { etc. }}{a_{0} a_{0}+\beta_{0} b_{0}+\text { etc. }} \frac{a_{0}\left(\frac{a_{0} a_{0}+a_{1} a_{1}}{a_{0}+a_{1}}\right)+\beta_{0}\left(\frac{\beta_{0} b_{0}+\beta_{1} b_{1}}{\beta_{0}+\beta_{1}}\right)+\text { etc. }}{a_{1}\left(\frac{a_{0} a_{0}+a_{1} a_{1}}{a_{0}+a_{1}}\right)+\beta_{0}\left(\frac{\beta_{0} b_{0}+\beta_{1} b_{1}}{\beta_{0}+\beta_{1}}\right)+\text { etc. }} \tag{28}
\end{equation*}
$$

Lehr's method uses the arithmetic average, firstly with double-weighting, secondly on the mass-units that have the same average price over both the periods compared. It is also unsatisfactory, the objection to the methods of both Drobisch and Lehr being that were an equality of prices at two periods accompanied by a large difference (increase) in mass-quantities, it would lead to a difference (increase) in the price-index. That is to say, though the price of every commodity might remain the same, the formulae both of Drobisch and Lehr would furnish different price-indexes.
11. Erroneously Alleged Defects in the Geometric Mean.-Laspeyres (a professor in the University of Basle) urged that the geometric mean, suggested by Jevons, was defective, supporting his contention by the following argument:-He supposes that from date 0 to date 1 the price of commodity $A$ advanced from 1 to 2 ,
and commodity $B$ declined from 1 to $\frac{1}{2}$. Since to purchase a unit of each commodity, 2 money-units would have been required initially, and at the second date $2 \frac{1}{2}$ moneyunits, he argues that the prices have advanced from 2 to $2 \frac{1}{2}$, that is $25 \%$. This, of course, is what is given by formula (2) herein, which limits the consideration to the case where the mass units purchased are constantly the same. In this case there can be no doubt as to which is the correct formula, in other words, the second aggregate of expenditure over the first aggregate is the only correct mode of computing the ratio of advance. But if, on the other hand, the general case is to be considered, where the degree of usage of each commodity may possibly have changed between the two periods, we cannot then assume that the mass-units are to be regarded as equal. The weights for price-ratios are expenditures, and in the case supposed by Laspeyres they are not equal. In this instance the "weights" at date 0 are the same for commodity A and B, but at date 2 the "weights " have materially changed. If we take the " weighting" into account, then the geometric mean of the weights will give results very approximate to those which Laspeyres claims should be given, and yet the case is not quite so limited as his was. The illustration confirms the view that in the general case, the geometric mean gives undoubtedly the better result, and Laspeyres' case does not really dispose of Jevons' argument : all it shews is that when price-ratios are used, proper weighting is no less important than in any other case, contrary to popular economic opinion. Thus by formula (2) we have the price-index 125 (Laspeyres' alleged correct value). But using geometric mean weights we get-

| Commodity. | Date 0. | Date 1. | Price Ratio. |  | Weights. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1 | 2 | $0 p_{1}=2$ | $u_{0}=1 ; u_{1}=2$ |  |
| B | 1 | $\frac{1}{2}$ | $0 q_{1}=\frac{1}{2}$ | $v_{0}=1 ; v_{1}=\frac{1}{2}$ |  |

Hence the result by the geometric mean, with geometric mean weights $=125.99$.
We see from this that Laspeyres' argument fails wholly, if as was originally pointed out, it is remembered that comparisons are invalid which take no account of those variations in the relative importance of commodities, which may be described as changes in the standard of expenditure, similar for example to changes in the standard of living or regimen. In other words, Laspeyres' contention that the geometric mean by Jevons' method gives no change of price-index, is merely a consequence of an inappropriate method of deducing a price-index, and confirms the view hereinbefore expressed, that exact " weights" must be used, if the deduced price-indexes are to have any economic value. Jevons' own suggestion, that perhaps the harmonic mean may be taken, is in general invalid, for reasons already indicated.
12. The Aggregate-Expenditure Method, the Best.-In § 10, Scrope's method, Laspeyres' and Paasche's variations of this method, and Scrope's own " emended variation" have already been referred to. Scrope used the same mass-units at different periods, i.e., he assumed a constant regimen. Laspeyres' variation, also based upon a constant regimen, was that he used the mass-units of the initial period, while Paasche used those of the final period. A geometric mean between the two (even an arithmetic mean is sufficient) is perhaps more accurate. These four formulae are all summed up by formula (2), Laspeyres using as mass units $a_{o}, \beta_{o}$, etc.; Paasche $\alpha_{m}, \beta_{m}$, etc.; the geometric and arithmetic means are $\sqrt{ }\left(a_{o} \alpha_{m}\right)$, etc., and $\frac{1}{2}\left(\beta_{0}+\beta_{m}\right)$, etc. They are satisfactory only for any two years to be compared, but the fundamental idea for perfectly unequivocal comparison for a series of years is the cost of a definite regimen for those years. Hence with the aid of statistics we must make such attempt as is possible to define a regimen that may be regarded as applicable to each of the years which are to be included in a comparison.* That is, we assume mean values for $a, \beta, \gamma$, etc. Thus we use formula (2) or (26) with these mean values for the mass units.

We shall first shew the substantial identity of the only reasonably accurate price-ratio method, formula (I1), with the aggregate-expenditure method, formula (2). Denoting the average values of the mass-units by $a, \beta$, etc., we may shew $\dagger$ that if $J$ denote the result by (11) and $I$ by (2), then
$\log J=2\left\{\frac{\alpha a x+\beta b y+\ldots}{\alpha a+\beta b+\ldots}+\frac{\left.\alpha a x^{3}+\beta b y^{3}+\cdots\right)}{\alpha a+\beta b+\ldots}\left(\frac{\alpha a x^{5}+\beta b y^{5}+\ldots}{a c+\beta b+\ldots}\right)+\right.$ etc. $\}$
$\log I=2\left\{\frac{\alpha a x+\beta b y+\ldots}{\alpha a+\beta b+\ldots}+\frac{1}{s}\left(\frac{a a x+\beta b y+\ldots}{a x+\beta b+\ldots}\right)^{3}+\frac{1}{b}\left(\frac{a a x+\beta b y+\ldots}{a a+\beta b+\ldots}\right)^{s}+\right.$ etc. $\}$
in which $\frac{a_{1}}{a_{0}}=\frac{1}{1} \frac{+x}{-x}: \frac{b_{1}}{b_{0}}=\frac{1+y}{1-y}$; etc.

[^7]In all ordinary cases $x, y$, etc., are small quantities. If we suppose them equal, the two expressions are identical. If they range in value from 1 to 10 , and 5 be their mean value, then the difference $\log J-\log I=-3_{n}^{n-1} \xi^{\xi^{3}}$ where $n$ is the number of commodities.

If we also suppose the average expenditures on the commodities to range from $I$ to 10 , then $\log J-\log I=$ about $0.56 \xi^{3}$ when $n=100$, and is never large. Remembering that in practical examples $\xi$ can never be' say $\frac{1}{3}$, in which case $\xi^{3}$ is Thr. it is easy to see that the two expressions are sensibly identical for any large number of commodities.

Since the price-index found from price-ratios, using the properly weighted geometric mean, is sensibly identical with the price-index found from the aggregate of expenditures, it is evident that results by umweighted means of price-ratios should be rejected, and further that the weights of price-ratios are very important.

The advantages of the method of aggregate expenditures, formula (2), may be stated as follows:-
(i.) It is incomparably superior to the unweighted price-ratio method if the mass-mits are at all near the true usage-quantities.
(ii) If the mass-units' are only approximately correct, small difierences in their value will not sensibly vary the result.
(iii.) One can instantly see in practical computation the influence of each term on the result, and thus estimate the effect of any uncertainties.
(iv.) It is the simplest possible of all methods, the precision of which entitles them to consideration.

Finally in this connection it may be said that, reverting to formula (20a) in $\$ 10$, the "weights" or expenditures $a a_{0}, \beta b_{0}$, can be made an average (or a probable average if we must estimate the future) and dividing these averages by $a_{0}$, $b_{0}$, we get mass-units, which must on the whole be satisfactory, and further by assuming oven an approximntely true regimen. far more exact results will be obtained than are furnished by an imperfectly weighted price-ratio combination.
13. Conclusion.-The conclusion of the whole matter may be stated as follows:-
(1.) The only accurate comparison that can be made between the purchasing power of gold at any two dates is one made on the basis of a definite series of commodities. The differences between different price-indexes shew that even an extensive series of commodities does not give a definite general result.
(ii.) For international purposes it is desirable that a standard-series of commodities should be adopted, and that this standard series should be used as a basis for all international comparisons.
(iii) That in connection with this series a definite method as to obtaining prices should be adopted so that the results in one country will be immediately comparable with the results obtained in any other.
(iv.) The prices of individual elements in this standard series should be weighted according to the mean usage of the whole of the countries included.
(v.) The weighted aggregate-price expressed in terms of some gold-unit (say £1) should be furnished, and the comparisons based upon the ratio of the weighted aggregate prices, that is, according to formula (2) herein. Such a comparison is perfectly accurate and definite, and there is no mathematical objection to the mode of computing it.
(vi.) In all cases where price-matios are used, each commodity in the tabular lists should have associated with it the weight-number used in the computation of the price-index, and these numbers should be in the ratios of the expenditures on the commodities. In comparing the price-index of one period with another these changes of weights should be taken strictly into account.
(vii.) Where the weights between the two periods differ but slightly, no appreciable error will arise by taking their arithmetic instead of the geometric means.
(viii.) Where the weights are very different, the geometric mean should be employed. The general aggregate should be computed on the principle of the geometric mean, i.e., the logarithms should be taken out of the price-ratios, multiplied by the mean-weight; the sum of these products, divided by the sum of the weights will be the logarithm of the result required.
(ix.) Comparisons of this character assume that the usage of the aggregate of the commodities is everywhere the mean adopted, and are, therefore, on this assumption very accurate, so far as the mere computation method itself is concerned.
(x.) It is easily seen that, for simplicity, the price-ratio method cannot compare with the aggregate expenditure method.
(xi.) A result obtained on the lines suggested can be regarded, however, only as an individual ground of comparison, viz., one of a purely international character, and its intrinsic value will depend upon the extent to which the whole series of commodities and assigned weights may be regarded as internationally significant.
(xii.) Even such a basis as this will, in the lapse of time, doubtless be subject to a progressive movement, and it may become necessary to alter periodically the list of commodities as well as to vary the weights assigned to each.
(xiii.) To the extent this alteration takes place the new price-indexes will not be directly comparable with the old, and a special investigation would be required to connect the two.
(xiv.) The international basis, moreover, will, in general, not be the best possible or most appropriate for the individual nations in the group.
(xv.) For national purposes it would not be difficult, however, to include other necessary items.
(xvi.) For practical convenience it is eminently desirable that the international group-result should be kept intact.
(xvii.) The suitable variations of weighting and inclusion of other commodities for national purposes can easily be managed through repetition of commodities with positive or negative weights, and the inclusion of other commodities with appropriate weights.
(xviii.) The same remarks apply mutatis mutandis in regard to the preparations of price-indexes for particular classes in a community, for it will be readily recognised that the purchasing-efficiency of money varies from class to class, and the idea that there is a general value can be regarded as correct only in so far as it may be conceived to apply to "an average individual" (l'homme moyen of Quetelet).
(xix.) The international comparison-basis would furnish the norm with which the price-index of each nation could be compared, and both it and the national price-index would furnish norms with which the results for different classes within the community could be compared.
(xx.) In view of the value of a properly computed price-index, the mere trouble of taking out logarithms of prices is a negligible quantity, and even this is unnecessary for the formation of price-indexes on an international basis.
(xxi.) Finally we may say that the aggregate of expenditures on a definite regimen is the only satisfactory method that is at all convenient from the standpoint of computation.

## APPENDIX IX.

## ON THE ESTABLISHMENT OF A BASIS FOR INTERNATIONAL COMPARISONS OF THE EXCHANGE-VALUE OF GOLD, AND VARIATIONS in the cost of living.

BY G. H. KNTBBS, C.M.G., F.R.A.S., F.S.S., etc., etc. Federal Statistician, Australia.

## SYNOPSIS.

1. Introduction.
2. On the selection of a list of commodities.
3. On the determination of the units and weights of the commodities.
4. Price-indexes deduced from aggregate-expenditures.
5. Price-indexes from price-ratios.
6. Proof that the method of price-ratios with a certain weighting is practically identical with that of aggregate-expenditures.
7. Invalidity of arithmetic mean.
8. Comparisons of price-indexes when alterations in the list of commodities or in the units adopted have been made.
9. Price-indexes when the number of commodities is greatly changed.
10. Effect of change of regimen.
11. Pseudo-continuity of price-indexes with progressive change of regimen.
12. Suggested lists of commodities and scheme of working.
13. Conclusion.
14. Introduction.-The financial and general relations of one nation with another are now seen to be so intimately connected, that all changes in human affairs must be discussed in their broadest aspects on an international basis. To do this effectually it is necessary that for all matters, subject to statistical analysis, mean-values should be established which, in virtue of their nature, may constitute norms for all comparisons, and for extensive generalisations. The standard of living, the habits, tendencies, and general character, the degree of civilisation, and the financial methods of the whole of the western civilised world, though divergent in details, tend more and more to closely approximate to each other, so as to constitute the world in some special sense an economic unity. For this reason economic norms occupy an important position among those which should be established. These will represent not merely the experience and characteristics of a particular nation, but of the whole aggregate of nations of which it is but an individual member, and which constitute the international solidarity, and among the economic norms, a series of numbers (price-indexes) which shall reveal the variations of the exchange value of the basis of the monetary system (gold), necessarily takes the place of highest importance. Reflexions upon the whole matter disclose the fact that we have arrived at that stage of world-development when it has become necessary to enlarge Quetelet's iden of the " average man" (l'homme moyen) to include the idea of a representative man of large groups of nations; indeed we must also create the idea of the "average nation" (la nation moyenne). This "average nation," its constitution and general
characteristics, will represent the entire western world and will constitute the proper norm for the study of the deviations of individual nations or lesser communities forming the combined group. It is evidently of the highest value for all comparative studies of national characteristics.

It will often be essential, or at least desirable, to compare smaller communities, within the nation to which they belong, not only with the international norms (the characteristics of the average nation), but also with the characteristics of the nation of which they form a part. By these two processes we may arrive at the highest form of generalised statistical knowledge.

What has been stated above may be regarded as the fundamental principle to be applied in the statistical methods of the western world as it is now constituted. It alone recognises the essential solidarity of that world, and that the significance of national variations from the international average, can be duly appreciated only by comparision with such average.

In this connection it may be observed that one of the most far-reaching elements among the relations of nations is that which touches the phenomena of the fluctuations in the exchange-values of commodities. The most general expression for this is in terms of money, viz., price, since money, being the medium of exchange for all commodities, has in consequence become the common measure of their value. Thus price, expressing inversely the exchange-value of the medium of exchange (gold) against any commodity, enables the exchange-relation between all commodities to be immediately deduced.*

It is convenient sometimes to follow, for certain purposes, the fluctuation in its exchange-value of the gold-unit rather than to follow the course of prices. In other cases, however, prices serve most readily for such comparisons as are required. Again, we may combine commodities to form a group and fix our attention on the varying quantities of gold necessary to purchase such given group. This idea we shall see is of the highest practical importance.

For all international comparisons of exchange-value it is self-evident that there must be a common basis in respect of the commodities selected to measure the variations of that value. Unless the basis be identical for each country, the results must necessarily be dabious; that is to say, it will become impossible to clearly distinguish between the extent to which differences in the exchange-value of gold are due to differences in the aggregate of commodities, or are due to other phenomena affecting the exchange-value, for example variations in the quantity of gold available for currency purposes, changes in the velocity of the movement of currency, or such changes as extensions or contractions of credit, etc., all of which are variations in the effective supply of the medium of exchange.

The common basis referred to, in order to be of real value, must be sufficiently extended and so weighted as regards its individual elements, as to represent the usage of the aggregate of the nations grouped, or what is the same thing, the usage of the "average nation." It is further necessary that this one basis should be maintained for the whole period which a particular scheme of unequivocal comparisons is designed to include.

Here, however, a difficulty arises. It is no less obvious that to maintain the reality of the comparison, the basis must change if the usage of mankind changes. A perpetually fixed basis would not represent " the usage of the average nation." It may, therefore, be admitted that any satisfactory basis will exhibit a slow progressive change in regard to the elements of which it is constituted, and the weights that must be assigned to those elements. The character of this secular variation and the question as to how the exchange-value of gold is to be estimated when the usage of the average nation has changed, must be specially investigated. To this we shall refer later, though it will form no part of the first question for our consideration. It may, however, be noted that since changes in the usage of the "average nation " will necessarily vary but slowly, and probably cannot be predicted with any

[^8]exactitude, results must ordinarily be elaborated on a basis lasting for a given period (say a decennium), that is, on a basis which will always be a little out of date. This, however, is unavoidable, and may readily be seen to constitute no serious difficulty.

The whole question thus resolves itself into the following, viz. :-
(i.) The formulation of a sufficiently extensive list of commodities in common usage among the different nations included in the international group ;
(ii.) The determination of the relative importance of these commodities either directly, or by an appropriate combination of the results for each nation, so as to ascertain the " average" importance for the whole group;
(iii.) The technique by means of which the general result is ascertained.

It would seem that the simplest way of determining the relative importance or " weight", therefore, from an economic point of view, depends upon two elements, viz., the aggregate-usage and the price. Thus, for example, if we divide the price of a commodity for any year by the price for some other year arbitrarily selected as a datum year, the quotient may be called the price-ratio of those years in respect of the commodity in question. Now this may be regarded as one of many possible measures (viz., through any other commodity) of the variation of the exchangevalue of the money-unit (gold). It is immedintely obvious that the relative importance of a series of these measures would depend upon the relative expenditure on each commodity. Hence in attempting to deduce a general estimate from a series of price-ratios, we should, in order to ascertain the weight which is to be assigned to each commodity, first have to ascertain the aggregate expenditure for the whole of the group of nations concerned, or else the average price of each commodity, and the aggregate number of units used of each commodity.

If, on the other hand, we intended to base our conclusions as to variations of exchange-value on a definite average regimen of so many units of each commodity, then we should need either to ascertain the number of units of each commodity in the regimen from direct statistics, or we should have to divide the international aggregates of expenditure by the international averages of price, to find the number of units. As already indicated, which course it is desirable should be followed, will depend upon whether the variation in the exchange-value of gold is to be evaluated from the aggregate cost of a particular regimen (i.e., of so many units of a definite series of commodities) or is to be deduced as some "weighted" mean of a series of price-ratios.

As regards the question of relative weights, it may be remarked that there is obviously no intrinsic relation between units, as say between a gross, of one commodity, a ton of a second, and a gallon of a third, and it is therefore evident that the only common measure of the importance is the money or exchange-value of the aggregate use of the commodity. This, however, is unfortunately variable, the variations of price themselves producing changes of "weight." The difficulties, howover, are not insuperable, for in general the " means" for a large aggregate vary relatively slowly. We may assume therefore that it is practicable not only to establish a list of commodities, but also to assign to the price-ratios of each a " weight "-number, expressing its importance in the entire group. It may be further noted that this series of "weight"-numbers must apply to limited periods (e.g., for a decennium), and may then be revised; and it is of course possible also that tho list of commodities must also be periodically revised. We can also decide on the average number of units of each used, that is, the quantities of each commodity in the average regimen.

When a list of commodities and the relative number of units of each used, or appropriate " weight" numbers are to hand, it is necessary then to decide upon a suitable arithmetical technique of comparison. The only unequivocal or perfect system of obtaining comparable results is to compute the aggregate-value of the whole series, from the number of units of each commodity corresponding to average usage, and the average price for the particular period (day, month, quarter or year, for example) which it is desired to compare; see formula (3) in $\S 3$, hereinafter. Since the arithmetical labour involved is by no means prohibitive, it may also be very clesirable to watch the characteristics of monthly or quarterly fluctuations in these aggregates, for example, in order to study the variations of exchange-value of gold within the year itself, and the mean of the results of any smaller period would furnish the requisite mean value for a longer one. For example, the mean of the four quarterly results would give the mean value for a year.

These aggregate values deduced from the whole series of commodities from the prices of each; and using the proper number of units of each, enable all necessary comparisons to be made with mathematical strictness. We may define this as the " method of aggregate expenditures." This method is unquestionably better than that of using price-ratios with weights. Any year may be made a datum, and references may be made forward or backward from that year without in any way vitiating the comparison; in other words, the process in this instance is always arithmetically consistent. Expressed as an algebraic formula, the process is as shewn hereinafter in § 3, see (3).

A method already referred to which has been largely used and which, if properly handled, is also fairly, but never wholly, satisfactory, is to deduce the price-index from price-ratios with appropriate weights. The average price of each commodity for some year is taken as a datum, and the price-ratios are ascertained by dividing the price for any other period by the price for the datum period. The quotient. usually multiplied by 100 , is the price-ratio of the latter date compared with former, When price-ratios are used, it may be shewn that the only proper mode of combination is what is known as the geometric. and this method is the only one used which is arithmetically consistent. To obtain the geometric mean each price-ratio is raised to the power indicated by the "weight," and the product of the whole of the priceratios, so raised, is a radicand of which a root, equal to the sum of all the weights, is to be taken. The indicated operation is very simply effected by taking out the logarithms of the price-ratios, multiplying each by its corresponding weight, and dividing the sum of these products by the sum of the weights.

Expressed as an algebraic formula, this last prescription is denoted by :-
(1)

$$
\begin{equation*}
\ldots \ldots I=\left(p^{u} . \quad q^{v} . \quad r^{w} . \text { etc. }\right) \frac{1}{u+v+w+\text { etc. }} \tag{1}
\end{equation*}
$$

or logarithmically-

$$
\begin{equation*}
\ldots \ldots \log I=\frac{u \log p .+v \log q+w \log r}{u+v+\text { etc. }} \tag{2}
\end{equation*}
$$

in which $p, q, r$, etc., denote the price-ratios of a series of commodities, $u, v, w$, etc., their "weights," based upon expenditures, and I the price-index required.

This process gives values very similar to the previous one, and is arithmetically consistent. Attempts have been made to obtain price-indexes by multiplying each price-ratio by the corresponding "weight" and dividing by the sum of the weights. Such a process, however, is arithmetically invalid, since it gives incorrect ratios between different years. In other words, it furnishes different results according as to whether we work from the calculated general result or from the original data. This is sufficient ground for excluding the method. We shall shew later on the nature of the arithmetical inconsistencies referred to, and it may here be stated that the extraordinary differences in the exchange-value of gold, indicated in the different series furnished by economists, shew that some better arrangement must be made if the price-indexes or index-numbers are to have any general validity, or are to be used critically. The most fruitful source of these differences lies in the fact that the lists of commodities are not identical, and are subject to different weightings.

On the Selection of a List of Commodities.-It is evident that, in order to be comparable at any two periods, a commodity must not have materially changed in character or quality. Certain commodities, for example, may give less trouble in this respect than others; for example, such raw products as may be regarded as sensibly uniform in quality, or manufactured products that do not materially change in quality. Metallic ingots, pig iron, etc., may be taken as a fair illustration of the former, sugar, flour, etc., of the latter. But even in regard to these, either differences of quality or arbitrary preferences may cause the exchange-values to range between very wide limits.

It is well to point out here that variations of exchange-value may by no means be wholly attributable to a general variation in the purchasing-efficiency of gold. For example, articles in which the cost of manufacture enormously varies* will tend to reflect conspicuously every variation in the rate of the remuneration of labour. The obvious reason of this is, that with raw materials the proportion which labour represents is usually very small as compared with what it represents in very highly elaborated products. $\dagger$

[^9]I.t is self-evident that, with such commodities, the governing element is the rate of remuneration for labour, and that the price of the commodity tends merely to reflect the variations of this element.

As a consequence of the operation of influences of this kind, it would seem that in an international inquiry, either as to wholesale or retail prices, all commodities in common usage, and of which the quality is comparable and identifiable, might probably be inchuded, but whether this be so or not will depend upon the fundamental purpose of the inquiry.

If, however, we were compiling a world-wide index-number, representing variations in the exchange-value of gold, it might probably be desirable not to include all commodities the qualities and characters of which are comparable and indentifiable. but merely those for which. in addition, world-markets exist. Thus it might be desirable to exclude all such vegetables, fruits, etc., the price of which would necessarily be governed mainly by local conditions. In a pure " cost-ofliving" comparison such commodities and their prices could not of course be excluded.

We have seen that variations in exchange-value are not wholly attributable to variation in the purchasing-efficiency of gold against ordinary commodities, excluding labour, and further, that the object of the measure of the exchange-value varies according to the characteristics in the group of commodities by means of which it is mensured.

It is clear from the considerations just indicated that the series of commodities should not only be individually identifiable in respect of character and quality, but should also be well selected from such point of view as is important, otherwise the derived results will be dubious, and it is here that the principal difficulty arises, though there is no escape from it.

It must be observed it the same time, however, that progress in the technique of industry indicates that we can push this principle too far, a good illustration of which would be the state of steel manufacture before and after the introduction of the Bessemer and the Siemens-Martens processes. Other examples that might be cited are sugar, chemical products, etc., in which there have been striking advances in quality. The advances in technology have led in many cases to marked improvement in the quality of the manufactured articles. Since, however, the use of the article, thus improved in quality, may be continuous, and the change in quality may proceed by imperceptible changes, a feature not uncommon with regard to textiles, for example, it is not always possible to take so exact an account of differences of quality as has been indicated as necessary.

Neglect of facts of this kind may easily betray one into an undue faith in priceratios, and into the false belief that price-ratios for aggregates are unequivacally valid measures of the variations in exchange-value of gold, whereas the truth of the matter is that changes in the exchange-value of gold have been confused in the general result with variations in the quality of the articles, and consequential changes in their cost, utility, or esteem values.

What has been said is sufficient indication that in the selection of a series of commodities for the international basis, extreme care will have to be exercised. I have suggested a series, and have indicated their weight numbers at the end of this Appendix. This is done merely tentatively and purely by way of suggestion. It is supposed that ench item in this series is identifiable with sufficient certainty to make the aggregate cost of the whole series reliable. It cannot be too distinctly borne in mind that the difficulty is not in any way got over by the use of price-ratios, as is sometimes supposed, but is only hidden so as to be less readily discerned.

The question of the significance of labour in the cost of commodities already referred to is worthy of special attention. We proceed to consider the matter.

The fact that commodities differ greatly in respect of the value of the raw material of which they are composed, and the amount of labour which has to be appliod to that material in order to give them their final form, suggests that regard should be had thereto in the scheme of classification.*

As between one commodity and another the ratio of the two varies greatly, and price will tend to reflect all variations in the remuneration of labour in proportion as the labour element in the production of the commodity is large.

[^10]From this it can be seen that it matters much whether the aim of an inquiry be to ascertain the efficiency of gold in respect to commodities as influenced by wages, or as not influenced by wages, and it is from this point of view that it may at once be seen how desirable it is that the list of commodities should be so divided as to furnish series shewing progressive amounts of labour applied to their production. In this way it will become possible to detect the influence upon price of ruling rates of wages. If, therefore, a whole series of commodities be divided into several classes, each class shewing progressively larger amounts of labour, then we shall have the material for discriminating between the purchasing efficiency of gold in regard to raw material and highly elaborated materials, and will have the data to ascertain how far demands for higher wages are merely equating themselves by a rise in prices. For if it were possible for the prices of commodities to rise throughout in the same ratio as wages, then there would be no advantage, the change would be merely a nominal one. It is from this point of view that one sees that, in so far as change of remuneration for labour results in increased prices, the advantage tends to become unreal, and is nullified, the money which is paid for labour giving to its recipient no advantage in purchasing the commodities which satisfy his needs.

It is evident that this matter is of eminent economic importance. If in making, finally, the comparison of the price-indexes of the successive series of commodities in which the element of labour is playing a more and more conspicuous part and in which consequently the influence of the remuneration of labour is more and more felt it turns out that the latter tends to closely correspond to variations in the rate of wages, then the obvious economic deduction is that the result is due to variations in the remuneration of labour. Should the variations completely correspond with change in the cost-of-living for the class represented, the effect of rise in wages will be completely nullified by the rise in the price of the commodities used.

It will be seen from these considerations that the divisions of the list of commodities should, as far as possible, be homogeneous with respect to the relative cost of naw material to labour in the production of the commodity. We may conclude, therefore, that so far as the selection of commodities is concerned, the following principles may serve as a guide, viz. :-
(i.) The commodities should be identifiable in respect of their essential characters.
(ii.) They should be largely used.
(iii.) The whole series should be divided roughly into groups, homogeneous with respect to the relative value of raw material, and labour applied to convert each commodity into its final form.
(iv.) Only commodities which find a world-market should be used for international comparisons. for variations in the exchange value of gold.
(v.) A supplementary list of commodities of local production are necessary if it be desired to determine such variations in the cost of living as may bo attributable to variations in the exchange value of gold.
3. On the Determination of the Units and Weights of the Commodities.-The unit by means of which different commodities are usually measured, may be volume or weight, or number of articles, etc. ; for example, in English measure, a gallon or a bushel, a pound or a ton, a gross or dozen, etc. All such quantities or units may be called mass-units, and the number taken for each commodity should bo in the ratio of their actual usage. It is instantly evident, however, that there is no intrinsic relationship between economic value and the mass-units of different commodities ; for example, between a carat, in the measurement of the precious stones, and a gallon in the measurement of spirits; in fact it is readily perceived that in the nature of the case there can be only one common measure for the relative economic importance of different commodities in question of variation of exchangevalue, and that is the product of the money-value of a unit, and the number of units used, or upon the relative aggregate expenditure on the commodity. As previously indicated, when we suppose the number of units used to be constant at any two dates for which a comparison is desired, the best-in fact the only exact-comparison is the ratio of aggregate expenditure at the compared date to the aggregate expenditure at the original date. If the number of units of each commodity were not constant, then any deduced price-index would be vitiated by what may be called change of regimen. For this reason, once we decide upon the size of the unit which is to be compared, the mass-weight-number of units of usage may be determined by dividing the total expenditure by the price, and it is to be assumed that this number of units is constantly used throughout the periods compared.

There is, however, a much simpler way of stating the whole matter, viz., the following :-

It is obvious that when we use price-ratios the actual size of the unit used disappears. For example, price per ton divided by price per ton is the same thing as price per pound divided by price per pound. This has led to an erroneous opinion that price-ratios get rid of the necessity for considering the size of the unit, and that the weights assigned to the price-ratios in any computation represent the relative importance of the commodities. The relative importance, however, is measured by the aggregate expenditure since the money-unit is the only common measure of economic value or exchange-value. If, therefore, relative aggregate expenditure on any commodity (i.e., the proportion of the expenditure on the commodity in question to the aggregate expenditure on all commodities) be equal on any two occasions, the combining "weight" of the commodity remains unchanged, in the computation of price-indexes from price-ratios.

From this point of view it becomes apparent that it is possible to compute a general variation in the exchange-value of gold with fair accuracy, although the regimen on successive occasions may have changed. Without doubt this fact has also given rise to the erroneous impression that price-ratios are to be preferred, and that they escape the difficulty about a constant regimen. It may be pointed out, however, that the basis of comparison should undoubtedly be the mean-weight between the two occasions, but to take this into account the arithmetical work of comparison is greatly elaborated and tends to become prohibitive. We shall return to this point later. It will suffice here to observe that a very much more convenient system could be adopted, by using units of quantity which can be regarded as representing the average use of all the nations in the international comparison.

If for the aggregate of nations a list, shewing the total expenditure upon the various items of a whole series of commodities during any definite period of time existed, this would represent the usage, and furnish the required number of units, or the mass-weights, the supposition being that that usage expressed the habit or the necessity of the people. It would indicate the economic weight that should be attributed to the individual item, by the ratio which expenditure on that item bore to the aggregate of all expenditures on the list. Futher, if, as is desirable, it were preferred to use numbers of "mass-units" of each commodity so as to form aggregates by summing the prices multiplied by these units to form totals for the dates to be compared (the ratio giving the price-index) then all that is necessary is to divide the international aggregates of expenditures by the international average prices. The quotients are the units required.

It may here be observed that questions of exchange-value are very properly dissociated from those of utility-value, esteem-value, and cost-value or other special measures of value, for many commodities obviously have esteem-values wholly out of relation to their cost-values; in fact, business-practice endeavours to create esteem-values so markedly above cost-values as to ensure large profits to the manufincturer or supplier. In the questions with which we are dealing, however, exchangevalue is the only value that need be considered.
4. Price-Indexes Deduced from Aggregate-Expenditures.-It has already been said that much of the technique in connection with the determination of variations in exchange-value practically involves the clouding of the real issue in generalities; that the comparison is unreal or dubious to the extent that the regimen has changed, and that the preference for price-ratios merely arises from the fact that the defect in the technique of computing price-indexes from them has been relegated to a position where it is not discernible. In order to bring the matter into clear relief, lot us take a very elementary case where only two commodities are under consideration, and observe exactly what takes place in different methods of combination. We shall denote the basic date by the suffix 0 attached to any quantity, and the second or later date by the suffix 1., the two commodities being denoted by A and B. We shall suppose the usage of these commodities at the two dates to be as expressed in the following schedule:-

| Commodity. | Date 0. |  |  | Date 1. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | Units. | Price. | Units. | Price. |  |  |
| A | 1 | $@$ | 3 | 1 | $@$ |  |
| B | 2 | $@$ | 4 | 3 | $@$ |  |
|  |  |  |  | 5 |  |  |

and let us use first the method of the ratio of aggregate expenditures which, expressed algebraically, is-

$$
(3) \ldots \ldots \ldots \ldots I=\begin{aligned}
& a a_{1}+\beta b_{1}+\gamma c_{1}+\text { etc. } \\
& a a_{0}+\beta b_{0}+\gamma c_{0}+\text { etc. }
\end{aligned}
$$

in which $\alpha, \beta, \gamma$, etc., are the number of units of each commodity used at each date, and $a, b, c$, are the prices of those units, the suffixes denoting the dates. We observe first of all that in the case considered there have been changes in both regimen and price, but to determine the variation in the exchange-value of gold we must eliminate the effect of change of regimen. Let us then first consider a comparison based upon a supposed constancy of regimen. Thus we may take into account three cases, viz., where the regimen at the second date is as at the first; where the regimen at the first date is as at the second; and where the regimen is the arithmetic mean of the regimens at the two dates. This will give us the result shewn hereunder as the effect of change of price, viz. :-
(i) the regimen consists of one unit of the first commodity, and two units of the second commodity on both dates;
(ii.) the regimen consists of one unit of the first commodity, and of three of the second on both dates; and
(iii.) the regimen consists of one unit of the first commodity, and two-and-a-half of the second commodity.
We thus get the following results :-
Regimen of date 0 ; (i.);

$$
\begin{aligned}
& \frac{1 \times 6+2 \times 5}{1 \times 3+2 \times 4}=\frac{16}{11}=1.4545 \\
& \frac{1 \times 6+3 \times 5}{1 \times 3+3 \times 4}=\frac{21}{15}=1.4000 \\
& \frac{1 \times 6+2 \frac{1}{2} \times 5}{1 \times 3+2 \frac{1}{2} \times 4}=\frac{18}{13}=1.4230
\end{aligned}
$$

Regimen an arithmetic mean of
that at each date; (iii.);
5. Price-Indexes from Price-Ratios.-Suppose now that we attempt to calculate such results by means of price-ratios. We have the following price-ratios for the two dates, (1)/(0), $p$ denoting price-ratio for commodity $A$, and $q$ denoting priceratio of commodity $B$.

Commodity A. Price Ratio. Commodity B. Price Ratio.

$$
p \quad=\quad \frac{6}{3}=2 . \quad q=\frac{5}{4}=1.25
$$

At the first date the aggregate expenditure was 11, of which 3 was on $A$ and 8 on B, therefore the relative importance for $A$ was $\frac{3}{1 T}$ and for $B \frac{8}{11}$. At the second date the aggregate expenditure was 21 , of which 6 was for $A$ and 15 on $B$, therefore the relative importance on the second occasion was ${ }_{2}{ }^{6} \mathrm{~T}$ or $\frac{2}{7}$ for $A$, and $\frac{5}{7}$ for $B$. Hence the arithmetic mean of the weights was-

$$
\begin{aligned}
& \text { For } A, \frac{1}{2}\left(\frac{3}{11}+\frac{2}{7}\right)=\frac{43}{154}=u, \text { say ; } \\
& \text { For } B, \frac{1}{2}\left(\frac{8}{11}+\frac{5}{7}\right)=\frac{111}{154}=v, \text { say. }
\end{aligned}
$$

Hence, working by the geometric means, we have-

$$
\begin{aligned}
& \log .\left\{\left(p^{u} q^{v}\right)^{\frac{1}{u+v}}\right\}=\frac{\frac{43}{154} \log .2+\frac{111}{154} \log .1 .25}{\frac{43}{154}+\frac{111}{154}}=\frac{23.7013}{154}=0.153904 . \\
& =l_{o g} \text {. of } 1.4253 \text {. }
\end{aligned}
$$

Now this last result is sensibly identical with what we found by taking the aggregate, and is nearly the mean of the results by suppositions (i.) and (ii.), viz., 1.4272. (If we take the geometric mean of the weights instead of the arithmetic mean we get:-

$$
\begin{gathered}
\text { For } A, \sqrt{ }\left(\frac{3}{11} \cdot \frac{2}{7}\right)=0.27914 \\
\text { For } B, \sqrt{ }\left(\frac{8}{11} \cdot \frac{5}{7}\right)=0.72075
\end{gathered}
$$

the sum of which is 0.99989 , and this gives 0.1538963 the logarithm of 1.4253 as before.)

Two things are obvious from the example furnished for the case of two commodities, viz. :-
(a) That with a large number of commodities the mean number of units used of each may be taken as a basis for computation of a price-index from the ratio of aggregate expenditures at any two dates, (formula 3); and also
(b) that price-ratios weighted in proportion to the average expenditure will yield an almost identical result.
This may be shewn formally by finding an algebraic expression for the difference ( $D$ )-
(4) $. D=\left(p^{u} \cdot q^{v} \cdot r^{w} . \text { etc. }\right)^{\frac{1}{u+v+w+\text { etc. }}}-\frac{a a_{1}+\beta b_{1}+\gamma c_{1}+\text { etc. }}{a a_{0}+\beta b_{0}+\gamma c_{0}+\text { etc. }}=H-K$, say.
in which $u=\frac{1}{2} a\left(a_{0}+a_{1}\right) ; v=\frac{1}{2} \beta\left(b_{0}+b_{1}\right) ;$ and $w=\frac{1}{2} \gamma\left(c_{0}+c_{1}\right)$.

## 6. Proof that the Method of Price-Ratios, with Weighting according to Average

 Expenditure, is Practically Identical with that of Aggregate-Expenditures.-Since the method of determining price-indexes from price-ratios is commonly supposed to possess some advantages through its apparent generality, and since also such opinion is not sound, it is not unimportant to shew conclusively that it yields sensibly identical results in all practical cases. This may be shewn formally by finding an algebraic expression for the difference $(D)$ above.We may put $a$ for $\frac{1}{2}\left(a_{0}+a_{1}\right) ; b$ for $\frac{1}{8}\left(b_{0}+b_{1}\right) ;$ etc.; and also $a_{0}=a$ $(1-x) ; a_{1}=a(1+x)$; and similarly throughout.

Then we have

$$
\frac{a_{1}}{a} / \frac{a_{0}}{a}=\frac{a_{1}}{a_{0}}=\frac{1+x}{1-x} ; \quad \frac{b_{1}}{b_{0}}=\frac{1+y}{1-y} .
$$

In all practical instances $p, q, r$, etc., do not differ greatly from unity, hence the quantities $f(1 \pm x)$, etc., can be expanded in convergent series. Thus we have to find the value of-

The values of $\log H$ and $\log K$ are :-
(5) . . . $\log H=2\left\{\begin{array}{l}a a x+\beta b y+\cdots \\ \alpha a+\beta b+\ldots\end{array} \frac{1}{3}\left(\frac{a a x^{3}+\beta b y^{3}+\cdots}{a a+\beta b+\ldots}\right)+\frac{1}{5}\left(\frac{a a x^{5}+\beta b y^{a}+\cdots}{a a+\beta b+\ldots}\right)+\right.$ etc.
$(5 a) \ldots . \log K=2\left\{\frac{a a x+\beta b y+\ldots}{a a+\beta b+\ldots}+\frac{1}{3}\left(\frac{a a x+\beta b y+\ldots}{a a+\beta b+\ldots}\right)^{s}+\frac{1}{6}\left(\frac{a a x+\beta b y+\ldots}{a a+\beta b+\ldots}\right)^{s}+\right.$ sto.

## Therefore-

(6) $\ldots . \log H-\log K=\frac{2}{3}\left\{\frac{\alpha a x^{3}+\ldots \text { etc. }}{a a+\ldots \text { etc. }}\binom{a \alpha x+\text { etc. }}{\alpha a+\text { etc. }}^{3}\right\}+\frac{2}{5}\{$ etc.-etc. $\}+$ ote

Thus the first and large term of the expressions for the logarithms of $H$ and $K$ agree, but the sewond and subsequent terms differ. The unequivocal condition. that these terms shall be equal is that the prices hevo all incrensed or all diminished in the same ratio, viz., that $x=y=z$, etc., in which case the second terms become $3 x^{3}$ in each case, the third $8 x^{5}$, and so on: that is, the two expressions are then equal throughout. We shall shew also that in all practical cases they are equal; and first we note that the quantities $a a, \beta b$, etc., are always positive, but that $x, y, z$, etc., may be all positive, all negative, or may not be all of one sign : the latter is ordinarily the case. The quantities being of the same order of magnitude, it is obvious that the difference between the terms would be greatest when they
are all of the same sign. We consider the case, therefore, where $a a=\beta b=$ etc., but $y=2 x, z=3 x$, etc., In this case the average value for $x, y$, etc., will be $\frac{1}{2}$ ( $\mathrm{n}+\mathrm{l}$ ) $x=\xi$ say, $n$ being the number of commodities. Consequently we shall have for the value of the two cubic terms-
$\frac{2}{3} \frac{a a x^{3} \Sigma^{n} n^{3}}{a a n}=\frac{1}{6 n}\{n(n+1)\}^{\dot{2}} x^{3}=\frac{4}{3} \frac{n}{n+1} \xi^{3}$ and $\frac{2}{3}\left(\frac{a a \Sigma_{1}^{*} n x}{a a n}\right)^{3}=\frac{2}{3} \xi^{3}$
The difference, therefore, in this instance is-

$$
\log H-\log K=-\frac{2}{3} \xi^{3} \frac{n-1}{n+1}
$$

which is $\frac{2}{3} \xi^{3}$ when $n$ is infinite, and is only about $2 \%$ short of $\frac{2}{3} \xi^{3}$ when $n$ is 100 .
If further, we suppose that $\beta b=2 a a ; \gamma c=3 \alpha a$, etc., and $y=2 x, z=$
$3 \boldsymbol{x}$, etc., as before, we shall have instead of the above-

$$
\frac{16}{45} \cdot \frac{2 n+1}{(n+1)^{3}}\left(3 n^{2}+3 n-1\right) \xi^{3} \text { and } \frac{16}{81}\left(\frac{2 n+1}{n+1}\right)^{3} \xi^{3}
$$

the difference of which is-
$\log H-\log K=\frac{\xi^{3}}{405} \cdot \frac{2 n+1}{(n+1)^{3}} \quad\left\{144\left(3 n^{a}+3 n-1\right)-80(2 n+1)^{2}\right\}$
which has a value of about $0.56 \xi^{3}$ when $n=100$.
In examples practically occurring we can never have the average value of $\xi$ as great as, say, $\frac{1}{5}$, viz., its value when all commodities have on the average advanced about $50 \%$. Hence $\xi^{3}$ is less than $7^{\frac{1}{3} 5}$, and in the three cases considered for 100 commodities, the difference would be 0 , or less than $\frac{1}{75}$ and $\frac{1}{2} \frac{1}{5}$ respectively. This is the difference in the two logarithms, but each is ordinarily the logarithm of a number somewhere near unity, and consequently represents approximately the ratio of the error itself.

It has thus been proved that $H$ and $K$ are sensibly equal in the circumstances of the case under consideration. It is obvious from this that, if the use of weighted price-ratios is deemed to be justified on the ground of any supposed generality in combining different measures of the exchange-value of money, then it follows from formula (6) that the method of ratios of aggregate expenditures, formula (3), is also valid. It is certainly the simpler to use, and computation of price-indexes is greatly facilitated by its use. This, however, while not unimportant, is not its chief merit, which is that the computer sees clearly what he is doing, while in the use of price-ratios it is by no means obvious that improper weighting vitiates the results. It is now seen that the method of price-ratios with inaccurate weights is sensibly equivalent to forming an aggregate with units which do not represent the actual average usage.

It has been already indicated that the weighted geometric mean of the priceratios is alone valid, and it has been shewn that the method of deducing priceindexes from the ratio of aggregates of expenditure, based on the use of a constant number of units, is substantially identical therewith. It now remains to consider the method of arithmetic means, not unfrequently used.

The following demonstration that even the weighted arithmetic mean is invalid is therefore not unimportant. That the unweighted mean is quite invalid can be readily seen to be a consequence of the preceding proof of the approximate identity of the aggregate expenditure and price-ratio methods of deducing price-indexes. But we shall now examine the question of the legitimacy of using a weighted mean in another way.
7. Invalidity of Arithmetic Mean.-Using the suffixes preceding and following $I$, the price-index, to denote the dates to which it applies, we have by the method of the arithmetic mean of weighted price-ratios

$$
(7) \ldots \ldots{ }_{0} I_{1}=\frac{u p+v q+w r+\text { etc. }}{u+v+w+\text { etc. }}
$$

Hence if we make date 1 the basis, and obtain the price-index for date 0 in relation thereto, we ought to obtain by the same process-
since each price-ratio is the reciprocal of the former, and this expression (7a) should equal the reciprocal of the preceding one, viz., (7) that is, if the method were arithmetically valid. But if this equality held wo should have-

$$
(u+v+w+\text { etc. })^{2}=(u p+v q+w r+\text { etc. })\left(u_{p}^{1}+v \frac{1}{q}+w \frac{1}{r}+\text { etc. }\right)
$$

We see, that so far as the sum of the squares of the quantities $u$, $v$, etc., is concerned, the two sides are identical, but so far as the products in pairs go the right hand sido is always greater than the left when $p, q$, etc., are not equal. Or, limiting the consideration to two price-ratios, we have to shew that-

$$
\frac{u}{p}+\frac{v}{u+q} \text { should equal } \frac{u+v}{u p+v q}
$$

if the method be arithmetically consistent. Multiply both expressions by ( $u+v$ ) ( $u p+v q$ ), we then have :-

$$
u^{2}+\left(\begin{array}{c}
p \\
q
\end{array}+\frac{q}{p}\right)^{u} u+v^{2} \text { should equal } u^{2}+2 u v+v^{2}
$$

that is, $p / q+q / p$ should equal 2. It, however, always exceeds that quantity unless $p=q$. The method of taking weighted arithmetic means, formula (7) is consequently arithmetically invalid, being irreversible; in other words, if priceindexes for a series of years are computed by the formula, they do not furnish the same ratios among one another as are furnished by the original data using the same process.

The weighted geometrie mean, on the other hand, formula (1), is consistent, and furnishes a series of price-indexes which furnish the same ratios as are furnished by the originel data.
8. Comparisons of Price-Indexes when Alterations in the List of Commodities or in the Units Adopted have been made. - It has already been pointed out that if price-indexes are to be strictly accurate, then change of regimen, that is to say, either a change in the actual list of commodities or in the units adopted for individual members of the list, cannot be admitted, otherwise variation in the exchange-value of gold becomes confused with the effect of change of regimen. The conception that now commodities may also furnish additional evidence of the exchange-value of gold is valid only when they belong to both periods to be compared. More definitely, if $a, \beta, \gamma, \delta$, etc., denote the numbers of units of the commodities $A, B, C, D$, etc., we cannot compare regimen $a \mathrm{~A}+\beta \mathrm{B}+\gamma \mathrm{C}$ etc., with say $\hat{\beta}_{1} \mathrm{~B}+\gamma_{1} \mathrm{C}+\delta_{1} \mathrm{D}$ etc., though we could of course, as already shewn, compare regimen $\beta \mathrm{B}+\gamma^{\mathrm{C}}$ with $\beta_{1} B+\gamma_{1} C$, the commodities $B$ and $C$ being common to both. Thus comparison can be made for example by assuming an arithmetic mean regimen, viz., $\left.\frac{1}{2} \beta+\beta_{1}\right) \mathrm{B}+\frac{1}{2}\left(\gamma+\gamma_{1}\right) \mathrm{C}+$ etc., to apply to the dates to be compared.

While the above statement is true, it is also true that the validity of any computation of price-indexes becomes of questionable value if the adopted list of commodities with assigned units of usage, (or price-ratios with their assigned weights) fails to coincide with the usage of the group of nations aggregated for international comparisons. The two things to be attended to are (i.) what may be called arithmetical validity, and (ii.) conformity to economic facts. From this point of view, it is to be regarded as inevitable that in the course of time changes will occur both in the commodities and their units of usage (or the weights assigned to their changes of price-ratios) in the international list. A revision, therefore, could perhaps be made every ten years, and the question then arises whether continuity in the ex-change-value relation can be established, and if at all, in what way.

Lot us suppose that, for one decennium, say. the comparisons heve been based npon $m$ commodities, and that then a change is made, and comparisons are afterwards based on $n$ commodities. Of these $m$ and $n$ commodities let us suppose also that there are $k$ common to each series; and moreover, that the units used (or the
series of weights assigned to price-ratios) used are not the same on the two occasions. We have already shewn that in such a case we can found a comparison only on some common regimen, preferably the arithmetic mean of the units used (or, if priceratios are used, the mean of the weights assigned to the prices of these $k$ commodities).

Primarily it is to be observed that strictly we can make a comparison only through the $k$ commodities constituting that part of the regimen common to the two periods. This is evident from the fact that change of regimen produces its own effect on the aggregate of a list of commodities, or on the weighted mean of price-ratios, the exchange-value of money being constant. And it is for this reason that, if we want to compare the exchange-value at any two periods we can do so only on some given number of units of a group of commodities existing at both periods; and to have the highest significance these assigned units of usage should, as near as possible, express the actual usage at either date, and hence may be taken as the arithmetical mean of the units at either date or of the weights used in connection with the price-ratios. For the method of aggregates the units may be the arithmetical means of the units used in either.*

It is obvious from this that there can be no real continuity in a series of price. indexes where the series of commodities used or the units of usage have changed, or where the weights assigned to the price-ratios of individual commodities have altered. For this reason, when a change of basis is made, the results should be computed on the old basis for the first year of the new series. Thus for this year the aggregates are formed on both bases, the one giving the closing value of the price-indexes, and their value is the factor to be used for the results given in the new series. The supposition, however, that by this process the second series of price-indexes is perfectly continuous with the old series is subject to some qualification, for the new series cannot strictly be referred back in this way. A perfect comparison between any two periods can be made only on the basis of the average usage of the series of commodities common to the two dates, the units assigned being a mean of the units assigned for the two dates.

To express the whole matter definitively, let $\Sigma_{0}, \Sigma_{1}$, etc., denote respectively the aggregates $\alpha^{\prime} a_{0}+\beta^{\prime} b_{0}+$ etc. $; a^{\prime} a_{1}+\beta^{\prime} b_{1}+$ etc. $; a^{\prime} a_{j}+\beta^{\prime} b_{j}+$ etc. ; the units $a^{\prime}, \beta^{\prime}$, etc., denoting the quantities regarded as constant throughout the first period (say a decennium). At the end of this period a change is made in the commodities and the units; viz., for the date denoted by $j$ ( $j$ would be 10 if the change were decennial), and $a^{\prime \prime}, \beta^{\prime \prime}$, etc., are the units used in the second period.

Then we can obtain an imperfect continuity of the exchange-values by forming the price-indexes according to the following scheme, viz. $:$ :-

$$
\begin{equation*}
\ldots \cdots{ }_{0} I_{1}=\Sigma_{1} / \Sigma_{0} ;{ }_{0} I_{2}=\Sigma_{2} / \Sigma_{0} ; \ldots{ }_{n} I_{j}=\Sigma_{j} / \Sigma_{0} ; \text { etc. } \tag{8}
\end{equation*}
$$

Then if for $\Sigma_{j}$ we form a second sum, using the new units and denote this by $\Sigma^{\prime}{ }_{j}$, we shall have-

$$
\begin{equation*}
\cdots{ }_{0} I_{j}=\Sigma_{j} / \Sigma_{0} ;{ }_{j} I_{j+g}=\Sigma_{j+j}^{\prime} / \Sigma_{j} ;{ }_{0} I_{j+!}=\left(\Sigma_{j} / \Sigma_{0}\right)\left(\Sigma_{j+g}^{\prime} / \Sigma_{j}^{\prime}\right) . \tag{9}
\end{equation*}
$$

in which $g$ denotes any year in the second period; or fully expressed :-

$$
\begin{equation*}
\ldots \cdot{ }_{a} I_{j+\theta}={ }_{0} I_{j} \cdot{ }_{j} I_{j+\theta}=\frac{a^{\prime} a_{j}+\beta^{\prime} b_{j}+\text { etc. }}{a^{\prime} a_{0}+\beta^{\prime} b_{0}+\text { etc. }} \cdot \frac{a^{\prime \prime} a_{j+}+\beta^{\prime \prime} b_{j+g}+\text { etc. }}{a^{\prime \prime} a_{j}+\beta^{\prime \prime} b_{j}+\text { etc. }} \tag{10}
\end{equation*}
$$

It is obvious from this last expression that any dissimilarity in the aggregate of the units of usage for the two periods does not prejudice the results, directly. Nevertheless it is equally obvious that the results of the second period are not strictily comparable with those of the first period. For the proper relation between any two results should be based on the mean number of units used for the two dates, and thus would be as follows :-

Let $a$ denote $\frac{1}{2}\left(\alpha^{\prime}+a^{\prime \prime}\right) ; \beta$ denote $\frac{1}{2}\left(\beta^{\prime}+\beta^{\prime \prime}\right)$; etc. Then the results for the year say 0 and the year $j+g$ should be

$$
\text { Correct result- } \quad{ }_{o} l_{j+g}=\frac{a a_{j}+a+\beta b_{j}+a+\text { etc. }}{a a_{0}+\beta b_{0}+\text { etc. }}
$$

The tabular results according to formula (10) would, however, differ from this. The measure of this difference we propose now to determine, and we consider first the case where the changes in the number of units of usage are relatively small, and where

[^11]the commodities are the same. In this case we may put $a^{\prime}=\dot{a}(1-x)$ and $a^{\prime \prime}=(1+x) ; \beta^{\prime}=\beta(1-y)$, and $\beta^{\prime \prime}=\beta(1+y)$, etc., then by interchanging the factors of the numerators the axpression (10) may be written-
\[

$$
\begin{equation*}
\ldots \frac{a(1+x)}{a(1-x)} \frac{a_{j}+j+\beta(1+y) b_{j}+!+\text { etc. }}{a_{0}+\beta(1-y) b} b_{0}+\frac{a(1-x) a_{j}+\beta(1-y)}{a(1+x) a_{j}+\beta(1+y)} b_{j}+\text { etc. etc. } . \tag{11}
\end{equation*}
$$

\]

$=\frac{a a_{j+g}+\beta b_{j}+a+\text { etc. }+\left(x a a_{j+a}+y \beta b_{j+g}+\text { otc. }\right)}{a a_{0}+\beta b_{0}+\text { etc. }-\left(x a a_{0}+y \beta b_{0}+\text { etc. }\right)} \times \frac{a a_{j}+\beta b_{j}+ө \text { tc. }-\left(x a a_{j}+y \beta b_{j}+\text { etc. }\right)}{a a_{j}+\beta b_{j}+\text { etc. }+\left(x a a_{j}+y \beta b_{j}+\text { etc. }\right)}$
If $S$ denote the sum of the quantities outside the brackets, and $s$ the sum of the quantities within the brackets, then this last expression may be written-

$$
\begin{gather*}
S_{j}+g+s_{j}+g  \tag{12}\\
S_{0}-s_{0}
\end{gather*} \quad S_{j}-s_{j}=\frac{S_{j}+g}{S_{j}+s_{j}}=\frac{\left(1+\frac{s_{j}+g}{S_{j}+g}\right)\left(1-s_{j}\right.}{s_{j}+g}\left(1-s_{0}\right)\left(1+S_{j}\right)\left(1-S_{j}\right)
$$

Now $S$ is a very large quantity compared with $s$, therefore $s / S$ is a very small quantity compared with unity, and consequently the right-hand factor in this last equation (with four brackets) must be very nearly unity. It can be seen somewhat more clearly if wo put

$$
\begin{equation*}
\ldots \ldots S=\frac{1}{3}\left(S_{0}+S_{j}+S_{j+q}\right) ; s=\frac{1}{3}\left(s_{0}+s_{j}+s_{j+g}\right) ; \tag{13}
\end{equation*}
$$

and also

$$
(13 a) \ldots \ldots S_{0}=S(1+\xi) ; S_{j}=S(1+\eta) ; S_{j+\jmath}=S(1+\xi) ;
$$

and

$$
\begin{equation*}
\ldots \ldots s_{0}=s(1+\chi) ; s_{j}=s(1+\phi) ; s_{j+0}=s(1+\psi) ; \tag{13b}
\end{equation*}
$$

so that we shall have

$$
(13 c) \ldots \ldots \xi+\eta+\xi=0 ; \text { and } \chi+\phi+\psi=0
$$

The expression (12) then becomes-

It is obvious that in this expression the whole of the terms denoted by Greek letters are srnall terms, and are also terms of the same order; and it is evident, therefore, that unless'prices or weights change very greatly the right-hand factor may be taken as unity.

It may be pointed out that in actual cases the quantities $S_{0}, S_{j}$, and $S_{j+g}$ are sensibly identical to the order of, say, several per cent. only; and $s_{0}$, $s_{j}$, and $s_{j+g}$ are Tusually very small; hence this factor in brackets will in general be so near unity as often to be satisfactory. In other words, the quantities $x a a, y \beta b$ etc., are of a much smaller order than $a, \beta b$, etc., and, entering into the result some with the + and others with the - sign, tend consequently to disappear in the final result.

It may be proper here to note that this last expression shews at once the advantage of the weights being so determined that, for the year on which the basis is changed, the aggregate of expenditures calculated with the two systems of weights shall be identical; for in such a case the values of $a^{\prime}$ and $a^{\prime \prime}, \beta^{\prime}$ and $\beta^{\prime \prime}$, etc., differ on the average the least possible. We may say finally that if the value of the right-hand factor of (12) (viz., that containing the four quantities in brackets) is unity then the continuity is satisfactory; if not, then it is unsatisfactory, and in proportion as it differs from unity : this expression or its equivalent (14) affords, therefore, the necessary criterion.

We shall see later that it is desirable that the units for the second period should be so determined that for the year of change $\Sigma_{j}=\Sigma_{j}^{\prime}$. As soon as the relative numbers $a^{\prime \prime}$, $\beta^{\prime \prime}$, etc., of the various units have been ascertained, this can readily be effected by multiplying these by an appropriate factor, $\kappa$, given by the formula

$$
\begin{equation*}
\kappa=\frac{\Sigma_{j}}{\Sigma_{j}^{\prime}}=\frac{a^{\prime} a_{j}+\beta^{\prime} b_{j}+\text { etc. }}{a^{\prime \prime} a_{j}+\beta^{\prime \prime} b+\text { etc. }} ; \text { that is } \kappa\left(a^{\prime \prime} a_{j}+\text { etc. }\right)=a^{\prime} a_{j}+\text { etc. } \tag{15}
\end{equation*}
$$

Thus we obtain a new set ( $a^{\prime \prime \prime}=\kappa a^{\prime \prime}, \beta^{\prime \prime \prime}=\kappa \beta^{\prime \prime}$, etc), proportional to those ascertained, viz., $a^{\prime \prime \prime} ; \beta^{\prime \prime \prime}$, etc. When this has been done, then the aggregate expenditure based on the corrected units for the second period is identical with the aggregate expenditure based on the units for the original period, notwithstanding that the system of units has been altered. That is, for the year of change the aggregate expenditure is unaltered, but the distribution among the commodities has been changed.
9. Price-Indexes when the Number of Commodities is greatly changed.-We now pass to the consideration of the case where only some of the commodities are common to the two series, and the weights on the occasions compared are very different. In such a case we can continuously trace an exchange-value relation only through the $k$ commodities common to the two groups, and the only theoretically satisfactory comparison is one where the two periods are compared on an identical basis, viz., the arithmetic mean (or more strictly on the geometric mean) of the two series of units. In practical examples it is probable that it is never necessary to use the geometric mean, for in all practical cases the change of regimen from decennium to decennium can hardly be such as to involve very great differences of weights, or even to involve the alteration of a very large number of commodities. The determination of relations of $k$ commodities of different weights in the series of commodities for the two periods will not therefore be unsatisfactory: In fact, it may be said that in almost every practical example the two means (arithmetic and geometric) will give practically identical results.

The reason of this is that the two means rarely differ very much, as will be seen from the following table, the original unit being l:-

| (a) Number of new units $N=1$ | 2 | 3 | 4 | 5 | 9 | 10 | 19 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (b) Arithmetic mean .. 1 | $1 \frac{1}{2}$ | 2 | $2 \frac{1}{2}$ | 3 | 5 | 5 즌 | 10 |
| (c) Geometric mean .. 1 | 1.41 | 1.73 | 2.0 | 2.24 | 3 | 3.16 | 4.36 |
| (b-c)/c Percentage of difference divided by $N$ : 0 | 3.03 | 5.16 | 6.25 | 6.83 | *7.41 | 7.39 | 6.81\% |

* Maximum value.

From this table it is seen that if the new units be $N$ times the preceding units the arithmetic mean will exceed the geometric mean by never more than $7.4 . N$ per cent. of the latter. Consequently whatever mode we assume for the growth of the unit from one value to another we may take the arithmetic mean of the units in practical examples.

Reverting to formula preceding, and remembering that the sum in these cases is for the $k$ commodities only, it will still be true that the product of the bracketed quantities in (12) and (14) will be sensibly unity for contiguous decennial periods. In this instance a kind of general continuity can be established even though the regimen is changed (it may be) per saltum each decennium. We proceed to elaborate the question. Whenever the number of commodities has been changed the question of continuity can be tested in the following manner, viz. :-

Let $R$ denote the computed aggregate of expenditure on the commodities which appear in the old but not in the new list; $S$ and $S^{\prime}$ denote the aggregate of expenditures on the continued commodities, viz., those appearing in both the old and new lists ; $T^{\prime}$ the aggregate of expenditure on those appearing in the new list only, and let as before the suffixes denote the year to which the expenditure refers. When the relative values of the units to be used for the new period (that is, for expenditure $S^{\prime}+T^{\prime \prime}$ ) have been found, then these units must be so corrected, see formula (15), that the expendit… ... the $k$ commodities whose aggregate is $S$ or
$S^{\prime}$ shall be identical, whether calculated by the new or by the old units. Then we shall have $\kappa S^{\prime}=S$, and $\kappa T^{\prime}=T$, so that $S$ is identical with either series of units, and ' $l$ ' is calculated on the corrected relative units (the correction making of course no change in their ratio to one another). Then we have by an operation similar to formula (10),

$$
\begin{equation*}
\ldots{ }_{0} I_{j+u}=\frac{S+R_{i}}{S_{0}+R_{0}} \frac{S_{i+a+T_{j+a}}=}{S_{j}+T_{j}}=S_{j+g} \quad S_{0} \quad\binom{1+\frac{R_{j}}{S_{i}}}{\frac{T_{j}}{S_{j}}} \cdot\binom{1+\frac{T_{j+a}}{S_{j+a}}}{1+\frac{R_{0}}{S_{0}}} \tag{16}
\end{equation*}
$$

Now, since $S_{j} / S_{o}$ is continuous under the old system of units, and $S_{j+g} / S_{o}$ is continuous under the new system ; $S_{\mathrm{j}+0} / S_{0}$ is at least what we have called psoudocontinuous through the entire period, this pseudo-continuity being attained by the correction of the units, so that the aggregate of expenditure on the $k$ commodities is identical with either system of units.

It can be seen from the above expression that if in introducing new commodities we take care that the aggregate expenditure on these, with corrected units, exactly equals the expenditure on those omitted at the year of change, we secure this. viz., that the left-hand term in brackets in formula (16) shall be unity, and further that the fractional terms on the right-hand term in brackets shall be of the same order and also in most cases sensibly equal. For this reason it is eminently desirable that the units be so determined that the whole aggregate of expenditure shall be identical with the now units as well as the aggregate for the commodities common to both groups. Then if the quantity in the right hand brackets is sensibly unity we can regard the pseudo-continuity as establishod. In practical examples $g$ should be one, that is the example should apply to the year immediately following that in which the change in the commodities and units is made.

Where it is desired to add a number of commodities such that the expenditure thereon is large as compared with expenditure on those omitted, we rewrite the terms in brackets in (16)

$$
\begin{aligned}
& \text { (16a) }
\end{aligned}
$$

In this $R_{j} / S_{j}$ is a quantity which is ordinarily nearly equal to $R_{0} / S_{0}$, and also $T_{j+g} / S_{j+g}$ is ordinarily comparable to $T_{j} / S_{j}$. When this condition happens to be satisfied the continuity may be satisfactory despite the fact that a relatively large addition of commodities has been made as compared with those omitted.
10. Effect of Change of Regimen. - When the product of the factors in (16) and ( $16 a$ ) is not unity, then they exhibit approximately the consequence of change of rogimen.

In connection with a cliscussion on the variation of the exchange-value of gold the effect of change of regimen is to be carefully distinguished from mere variation in the magnitude of the units. It can best be illustrated thus :--

Suppose that, with the same list of commodities for any datum year, and using two series of units, we have equal expenditures, agreeably to the prescription of formula (15), and find with the prices for any other year a difference of expenditure, this difference measures the effect of change of regimen. To express this otherwise suppose that $I$ and $I^{\prime}$ denote the price-index as deduced with an identical list of commodities but with two series of units, of which let us assume $I$ is on an original. and $I^{\prime}$ on a new basis, the expenditures being identical for the datum year. Then we have for $\rho$ the effect of the change of regimen.
(17) $\ldots . \rho=I^{\prime} / I$.

Each year will, of course, give a different value for $\rho$, but if actual results shew that the variations of $\rho$ are very small, we can regard the (weighted) mean as furnishing at general measure of the effect of the change. As the distance in time increases from the datum year, the individual values obviously become of less weight. Hence we may empirically adopt some such formula as

if we have the values of this factor for successive years $1,2,3 \ldots \ldots n$. In general the variations of $\rho$ will exhibit variations, shewing no definite tendency; when this is not the case the progressive change may demand special investigation.
11. Pseudo-Continuity of Price-Indexes with Progressive Change of Regimen. -

For comparative purposes stretching over long periods of time it would appear on the whole desirable to adopt a method, which would be sensibly accurate for short periods of time from the standpoint of the exchange-value of gold, and yet nevertheless represent for long periods the combined effect of change of regimen and altered exchange-value or purchasing-efficiency of gold, the change of regimen corresponding to variations in the international usage from period to period. Under such a system the ratio of price-indexes for distant dates would, strictly speaking, then cease to represent changes in the exchange-value of gold but rather those changes as modified by an alteration of the average regimen. Comparisons from the standpoint of variations in the exchange-value of gold alone would have to be dealt with by special investigations where necessary. We proceed now to consider the question.

The fundamental idea on which a pseudo-continuity can be developed is that for the years of change (constituting what we shall call the successive control years), the change of units shall be so controlled that the aggregate of expenditure on the $k$ commodities, common to the two groups, shall be identical with the two series of units (formula 15). This gets rid, in probably the most convenient way, of the difficulty that in general we cannot ascertain the absolute, but only the relative, number of units used of each commodity.

It will facilitate the explanation to describe the method schematically, and the method can best be illustrated as follows :-
Commodities disappearing.
Commodities constant to both periods.
Commodities being introduced.

ABC DEEGH——_I K M

Let 1900 be the last year when commodities say $A$ to $H$, are to be fully included. It is decided in 1910 to revise the list so that it shall contain commodities D to M , but not A to C. In this case 1901 is to be regarded as the change year. For this year we must see that the aggregate of expenditure on D to H is equal as required by formula (15) ; and must see also that, using the old units for A to C, the aggregate of expenditure is equal (approximately) to that on D to M working with the corrected units. When this has been done we decrease the units of A, B, C, yearly by onetenth of the original amount, and increase those of $\mathrm{I}, \mathrm{K}, \mathrm{L}, \mathrm{M}$, yearly one-tenth of their weight for 1900 , according to the following scheme, viz. :-

| Units for Commodities. | Factor corresponding to year. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1900 | 1901 | 1902 | 1908 | 1909 | 1910 |
| a, $\beta, \gamma^{*}$ | 10 | $\frac{9}{10}$ | $\frac{8}{10}$ | $\mathrm{i}^{2} 0$ | $\frac{1}{10}$ | -10 |
| ८, к, $\lambda, \mu \dagger$ | $\frac{0}{10}$ | ${ }_{10}^{10}$ | $\frac{1}{10}$ | $\frac{8}{10}$ | $\frac{9}{10}$ | $\frac{10}{10}$ |

Thus in this scheme $A, B, C$, have entirely disappeared in 1910 , and $I, K, L, M$, have appeared with their full values in the same year, while intermediately one series is increasing and the other is diminishing. We also change each of the units for the commodities D to $H$ one-tenth of their difference yearly so that the new values are reached in 1910. That is, if $\delta^{\prime}$ denote the corrected weight in 1910, and $\delta$ the weight in 1900 , the weight for $1900+n$ years ( $n$ being less than 10 ), will be

$$
\begin{equation*}
\ldots \ldots \ldots \delta+\frac{n}{10}\left(\delta^{\prime}-\delta\right)=\frac{10-n}{10} \delta+\frac{n}{10} \delta^{\prime} \tag{19}
\end{equation*}
$$

A very simple numerical illustration will shew the effect of the process, and for this purpose we neod take only two commodities which we may suppose to represent the continuous series. These will illustrate the nature of the difference of the two mothods. Let us then suppose a regimen of commodities in the first years of a series to be in the ratio of 1 of $A$ to 2 of $B$ : and for the fifth year to be 2.7 of $A$ to 2.4 of B.

We first find by formula (15), see hereinbefore, that with the prices as at the final or control year-

$$
\begin{array}{ccc} 
\\
- & 2 \\
3
\end{array} \quad \text { viz., } \quad \begin{gathered}
1 @ 4+2 @ 8 \\
2.7 @ 4+2.4 @ 8
\end{gathered}=\frac{20}{30}
$$

Fence the units become 1.8 and 1.6 , that is-

$$
1 @ 4+2 @ 8=1.8 @ 4+1.6 @ 8
$$

We thas obtnin the results in the table hereunder, viz. :-
(i.) for the method of continuously depending upon the original number of units of $k$ commodities, and
(ii.) for the method of changing the units yearly, respectively-


It is easy to see that the control which ensures the identity of the final aggregates (i.) and (ii.) for year 5, ensures also that the intermediate values for years 2,3 and 4 shall substantially agree. Similarly, since for the change-year the expenditure on the commodities added is to' balance that on those subtracted, we shall get a satisfactory continuity through that year, and thus results which shew the leffect mainly of change of price, though modified slightly by change of regimen.
12. Suggested List of Commodities and Scheme of Working.-The following table shows the commodities included by various authoritios in compiling Tndex-Numbers for difforent countries. In this tabular statement only commodities which are common to more than three of the 27 index-numbers have been included; commodities which are included in only one or in either two or or three of the index-numbers are specified in the notes at the end of the table. Where any commodity is included in more than three of the index-numbers the fact is indicated by a cross (X) ; in every case where more than one grade or quality of any commodity is included the small number shewn in brackets after the crose specifies the number of grades or qualities. Take in Table.

| Commodity. | Great Britain. |  |  |  |  |  |  | Germany. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| No. ofCommodities | 39 | 22 | 20 | 22 | 39 | 39 | 45 | 48 | 47 | 22 | 47 | 114 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Textiles, etc. | X |  |  |  | X | $\mathrm{X}(2)$ | X | X |  |  | . | X |
| Leather | , | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | X | X | A | $\underline{\mathbf{X}}$ |  |  | $\because$ | X |
| Tallow | $\because$ | $\frac{\mathrm{x}}{\mathrm{x}}$ | $\frac{\mathrm{x}}{\mathrm{x}}$ | ${ }^{\mathrm{X}}$ | X | $\underline{X}(2)$ |  |  | X |  | X | X |
| Cotton, Raw | $\ldots$ | $\mathrm{X}(2)$ | X | $\underset{\mathrm{X}}{\mathrm{X}}$ (2) | X(3) | X(2) | x | X | X | x | X |  |
| ", Cloth Yarn .. |  | X |  | X |  |  | $\cdots$ |  | $\ddot{\text { x }}$ |  | X | $\mathrm{X}(3)$ |
| Flax ${ }^{\text {F }}$. | X | X | $\underset{\text { x }}{ }$ | X | $\underset{\mathrm{X}}{\mathrm{X}}$ | $\underline{\mathbf{x}} \mathbf{( 2 )}$ | $\ddot{\mathrm{x}}$ | X | X | $\ldots$ | X | X |
| Hemp .. .. | . | X | X | X | X | X ${ }^{\text {(2) }}$ | $x$ | X |  | . |  | X |
| Jute ${ }^{\text {Linen }}$. $\quad$. | - |  |  |  |  | X | $\chi$ | $\cdots$ | $\dot{\mathrm{x}}$ |  | $\ddot{\mathrm{x}}$ | $\dot{\mathrm{X}}(\underline{2})$ |
| Silk, Raw |  | $\ddot{X}$ | $\ddot{\mathrm{x}}$ | $\ddot{\text { x }}$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | $\because$ | X | $\ddot{\mathrm{x}}$ | X | X |
| Wool | X | X | $\underline{\mathrm{x}}$ | X | $\mathbf{X}$ | X(3) | $\mathbf{X}(2)$ | $\dot{x}$ |  | .. | X | $\underset{ }{X}$ |
| Woollen Yarn.. |  |  |  |  |  |  |  | . | X |  | . | $x(2)$ |
|  |  |  |  |  |  |  |  |  |  |  |  | X |
| Beans .. .. | X | $\cdots$ | $\cdots$ | $\cdots$ | X | . | .. |  |  | . |  | X |
| Clover .. .. |  | $\cdots$ | $\cdots$ | $\cdots$ | X |  | $\cdots$ | X | X | $\cdots$ | X |  |
| Hay .. .. | X | $\cdots$ | $\because$ | $\cdots$ | X |  |  | . | . | $\cdots$ | $\cdots$ | $\cdots$ |
| Linseed | . | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | X | X | $\cdots$ | $\cdots$ |  | $\because$ |  |
| Maize | $\ddot{\mathrm{x}}$ | $\because$ | $\because$ | $\cdots$ | $\ddot{\mathrm{x}}$ | X | $\underset{\mathrm{X}(2)}{ }$ | $\ddot{\mathrm{X}}$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ |
| Peas . | X | $\cdots$ | $\cdots$ | $\cdots$ | X | .. |  |  |  | . |  | - |
| Rape Seed | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | . | - | $\ddot{x}$ | X | X |  | X | $\frac{\lambda}{\mathrm{X}}$ |
| Rice .- | $\ddot{x}$ | $\because$ | $\because$ | $\because$ | $\ddot{\mathrm{x}}$ | - |  | $\underset{X}{ }$ | $\frac{\mathrm{X}}{\mathrm{X}}$ | $\underset{\sim}{X}$ | X | X |
| Straw |  |  |  |  | X |  |  |  |  |  |  |  |
| Wheat - | X | x | x | X | X | $\mathrm{X}(2)$ | $\mathrm{X}(2)$ | X | X | X | X | X |
| Dairy Produce. <br> Bacon |  |  |  |  |  |  |  |  |  |  |  |  |
| Bacon Butter | . | $\because$ | . | $\because$ | X | $\frac{\mathrm{X}}{\mathrm{X}}$ | $\lambda$ | x | $\ddot{X}$ | $\because$ | $\ddot{\mathrm{X}}$ | $\ddot{\mathrm{x}}$ |
| Cheese | x | $\ldots$ | $\ldots$ |  | $\ldots$ |  |  | $\underline{X}$ | . | $\cdots$ |  | X |
| Bgas | X | $\ldots$ | $\ldots$ |  | $\because$ |  | $\underline{x}$ | .. | $\cdots$ | $\cdots$ | . | X |
|  |  |  |  |  |  |  |  |  |  |  |  | X |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| ${ }_{\text {Cocor }}$ Almonds | X | $\because$ | $\because$ |  | $\because$ |  |  | $\frac{\mathrm{x}}{\mathrm{X}}$ | X |  | $\underset{X}{X}$ | X |
| Coffee $\cdots \quad \cdots$ | $\because$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | $\because$ | $\dot{\mathbf{x}}(2)$ | $\mathbf{X}$ | $\overline{\mathrm{X}}$ (3) | X | X | X | X |
| Currants |  | $\cdots$ | . | . | . |  | .. | X | X | .. | X | X |
| Flour | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | X | $\because$ | X |  | $\cdots$ | . | X |
| Hops | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | X | $\because$ | ${ }^{\mathbf{X}}$ |  | $\ddot{\mathrm{X}}$ | X |
| Mart | X | $\ldots$ | . |  |  | $\cdots$ | $\because$ | $\cdots$ |  |  |  | X |
| Pepper | . | . | $\cdots$ |  | X |  | $\ldots$ | X | X | $X$ | X | X |
| Potatoes |  | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | X |  |  |  | $\cdots$ | x | X |
| Raisins. | X | $\cdots$ | $\cdots$ | $\because$ | $\cdots$ | . | $\cdots$ | X | 入 | $\cdots$ | X | $\stackrel{ }{ }$ |
| Rye Flour | X | $\cdots$ | $\cdots$ | $\cdots$ | $\because$ | $\cdots$ | $\ldots$ |  | $\cdots$ | $\cdots$ | $\cdots$ | X |
| Spirits . | $\mathbf{X}$ | $\because$ |  |  | $\ddot{\mathrm{x}}$ |  | X | $\dot{\mathbf{x}}(2)$ |  |  |  | $\underline{X}(3)$ |
| Sugar . | $\because$ | X | X | X | X | $\underline{\text { x }}(3)$ | X | $\mathrm{X}(2)$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | X | $\mathrm{X}(2)$ |
| Tea | $\cdots$ | $\frac{\mathrm{X}}{\mathrm{X}}$ | X | X | X | X(2) | X | X | X | X | X | X |
|  |  |  |  |  |  |  |  |  | $\cdots$ | $\cdot$ | $\cdots$ |  |
| Beef. | $\cdots$ | X | X | X | X | $\mathrm{X}(2)$ | X | X | . | $\cdots$ | $\cdots$ | X |
| Mutton .. | $\ldots$ | $\cdots$ | . | $\cdots$ | X | $\mathrm{X}(2)$ | X |  | . | $\cdots$ | $\cdots$ | $\frac{X}{X}$ |
| Pork Sheep |  | $\because$ | $\because$ | $\because$ | X | X | . | X | $\cdots$ | $\cdots$ | $\cdots$ |  |
| Fish : | $\underline{X}$ | $\because$ | $\because$ | $\cdots$ | $\cdots$ | $\ldots$ | x | X | X | $\cdots$ | X | $\dot{\mathbf{x}}(2)$ |
| Building Materials. |  |  |  |  |  |  |  |  |  |  |  |  |
| Bricks .. .. | $\ldots$ | x | X | X |  |  | X |  | $\cdots$ | $\ldots$ | - . | $\cdots$ |
| Miscellaneous. | $\cdots$ | X | X | X | X | $\mathrm{X}(2)$ | X | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| Caoutchouc |  |  |  |  |  |  | X |  |  |  |  | X |
| Indigo .. | . | x | X | X | X | x. |  | X | X | X | x | $\frac{\mathrm{X}}{\mathrm{x}}$ |
| Oils | $\cdots$ | X | X | X | $\mathbf{X}(2)$ | X (3) | $\dot{\mathrm{X}}$ (2) | X | $\mathrm{X}(4)$ | $\mathrm{x}(2)$ | X X (5) | $x(5)$ |
| Sodit ${ }_{\text {Saltetre }}^{\text {Sol }}$ | .. | $\cdots$ | $\because$ | $\because$ | $\cdots$ | X | $\cdots$ | X | X | X |  | X |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

various Investigations．

|  | ermany． |  | $\left\|\begin{array}{c} \text { A'striat } \\ \text { Hun- } \end{array}\right\|$ | France． |  |  | 1taly． | Switz－ erland． | India． | U．S．of America． |  | C＇nada | Australia． |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | － |
| 17 | 37 | 40 | 9 | 22 | 38 | 40 | 6 | 20 | 45 | 223 | 257 | 230 | 80 | 46 |
|  |  | $\underline{x}(2)$ | $\cdots$ | X | X | $x(2)$ |  |  | N | $\mathbf{X}(8)$ | $\mathbf{X}(7)$ | X（3） | N |  |
| $\ldots$ | X | x | $\cdots$ | N | － | $x$ | ． | $\cdots$ | $\cdots$ | $\mathbf{X}(2)$ | X | ${ }^{1}$ | X | $\because$ |
| $\cdots$ | ． | $\underline{x}(2)$ | $\cdots$ | － | － | x（2） |  | $\because$ | $\because$ | X | X 4 | X | $\underline{x}(7)$ |  |
| $\cdots$ | $\cdots$ |  | $\cdots$ |  |  | X | $\because$ | $\cdots$ | $\because$ | X（2） | $\frac{\mathrm{x}}{\mathbf{x}}$ | $\stackrel{\lambda}{\lambda}$ | X（2） | $\cdots$ |
| $\because$ | $\ddot{\mathrm{X}}$ | $\ddot{\mathrm{x}}$ | $\cdots$ | $\because$ | \％ | $\dot{\mathrm{x}}$ | $\because$ | $\cdots$ | $\because$ | $\because$ | N | X | $\because$ | $\cdots$ |
| ．． | ． |  | ． | x | X | ． | $\ldots$ | ． | $\cdots$ | X | $\mathrm{X}(2)$ | X | X | ． |
| $\cdots$ | $\because$ | $\underline{x}(2)$ | $\because$ | X | $\mathrm{X}(2)$ | $\mathrm{x}(2)$ | $\cdots$ | $\because$ | $\frac{\mathrm{X}}{\mathrm{X}}$ | X $\times(2)$ | X | X | $\dot{\mathbf{x}}(3)$ | $\because$ |
| $\because$ | $\because$ |  | $\because$ | X |  | X |  | $\cdots$ | $\underset{ }{\chi}$ |  | ${ }_{\mathbf{X}}^{\mathbf{X}}$ | $\stackrel{\rightharpoonup}{\mathbf{X}}$ | 2（3） | $\because$ |
| $\ddot{\mathrm{X}}$ | $\ddot{\mathrm{x}}$ | $\ddot{\mathrm{x}}$ | $\because$ | X | $\ddot{X}$ | N | $\because$ | $\because$ | X |  | X | X | $\ddot{\mathrm{X}}$ | $\because$ |
| $\because$ | $\because$ | X | $\because$ | $\because$ | $\stackrel{\mathrm{X}}{\mathrm{X}}$ | X |  | $\because$ | X | $\mathbf{X}(3)$ | X | X | $\cdots$ | $\because$ |
| $\because$ | $\because$ | 入 | $\because$ | $\because$ | X | X | $\because$ | $\because$ | $\cdots$ | $\cdots$ |  | ．${ }^{\text {．}}$ | $\because$ | $\because$ |
| ． | $\because$ | N | $\because$ | $\because$ | $\because$ | X | $\because$ | $\because$ | $\ddot{x}$ | $\cdots$ | r | x | $\cdots$ | $\cdots$ |
| $\because$ | $\cdots$ | ＊ | $\because$ | $\because$ |  |  | $\because$ | $\because$ |  |  | $\frac{\mathrm{X}}{}$ | $\underset{X}{ }$ |  |  |
| $\cdots$ | $\ldots$ | $\dot{x}$ | $\cdots$ | $\stackrel{\mathrm{X}}{\mathrm{x}}$ | $\dot{\mathrm{x}}(3)$ | $\dot{x}$ | $\because$ | $\cdots$ | $\ddot{\mathbf{x}}$ |  | $\frac{\mathrm{x}}{\mathrm{N}}$ | $\hat{\lambda}$ | － | $\because$ |
| ． | $\cdots$ | X（2） | $\ldots$ | N | N | $\mathrm{X}(2)$ | $\cdots$ | $\cdots$ | X | $\mathbf{X}(2)$ | X | X | X | ． |
| ， | ． | ， | $\cdots$ |  |  | ． | $\cdots$ | $\cdots$ | $\cdots$ | － |  | X | ． | $\cdots$ |
| X | X | X | x | $\cdots$ | $\cdots$ | X | $\ldots$ | X | $\cdots$ |  | $x$ | $x$ | $\mathrm{X}(2)$ | $\ldots$ |
| X | $\cdots$ | $\cdots$ | ． | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | X | $\cdots$ | ．． | $\cdots$ | $\cdots$ |
| $\ddot{\mathrm{x}}$ | $\cdots$ | $\because$ | － | $\because$ | $\cdots$ | ． | $\ldots$ | $\ddot{X}$ |  | $\because$ | $\ddot{\mathrm{x}}$ | $\ddot{x}$ | $\ddot{\mathrm{X}}$ | $\because$ |
| ． | $\ldots$ |  | － | $\because$ | $X$ |  | ＊ | $\cdots$ | X | $\cdots$ | ． | $\times$ |  |  |
| $\ddot{x}$ | $\ddot{\mathrm{x}}$ | $\stackrel{\rightharpoonup}{\mathrm{X}}$ | X | $\because$ | $\because$ | $\underset{\chi}{\chi}$ | $\lambda$ | $\ddot{\mathbf{X}}$ | $\cdots$ | $\because$ | $\ddot{\mathrm{X}}$ | X | $\hat{\mathrm{X}}$ | $\because$ |
| X | ．． | $\cdots$ | $\cdots$ | ． | $\cdots$ | ．． | ． | ． | $\because$ | $\cdots$ | ． | $x$ | X | $\cdots$ |
| $\cdots$ | $\ddot{\mathrm{X}}$ | $\ddot{\mathrm{x}}$ |  | $\ddot{\mathrm{X}}$ | $\ddot{\mathrm{X}}$ | $\dot{\chi}$ | ス |  | X | $\ddot{X}$ | $\ddot{\mathrm{x}}$ | $\dot{\chi}$ | $\ddot{\mathrm{X}}$ | $\ddot{\mathrm{x}}$ |
| x | ． | X | X | ．． | ． | X | ．． | $\pm$ | ．． | ． | X | X |  | ． |
| $\mathbf{X}$ | $\ddot{\mathrm{x}}$ | $\ddot{x}$ | $\dot{\lambda}$ | X | X | $\dot{\mathrm{X}}(\underline{2})$ | X | X（2） | $\ddot{\mathrm{x}}$ | $\cdots$ | $\stackrel{\mathrm{X}}{\mathbf{X}}$ | $\frac{\lambda}{x}$ | X |  |
| X |  |  |  |  |  |  |  | X |  |  |  |  |  | $\mathrm{X}(2)$ |
| X | $\ldots$ | $\mathbf{X}$ | $\ldots$ | N | $x$ | $\mathbf{X}$ | $\ldots$ | X | X | X | X | X | X | X |
| $\cdots$ | $\cdots$ | $\cdots$ | $\because$ | $\cdots$ |  | $\cdots$ |  | $\ddot{\mathrm{X}}$ | $\cdots$ | X | X | $\frac{\mathrm{X}}{\mathrm{X}}$ | $\frac{\mathrm{X}}{\mathrm{X}}$ | X $\mathbf{X}$（2） |
| $\ddot{\mathrm{x}}$ |  | ． | $\because$ | $\because$ | $\cdots$ | $\because$ |  |  | $\because$ | X | X | X |  | X |
| ． | ． | ． | $\cdots$ | $\cdots$ | $\ldots$ |  | $\cdots$ | $\cdots$ | $\cdots$ | ． | $\cdots$ | ． |  | $\cdots$ |
| $\cdots$ | $\ddot{\boldsymbol{x}}$ | X | $\because$ | X | $\ddot{\mathrm{x}}$ | X | $\because$ | $\because$ | $\ddot{\mathrm{x}}$ | x | x | $x$ | $\mathbf{X}$ | $\cdots$ |
| ． |  | ． | ． | $\cdots$ | ． |  | ． | ． | ． | X | X | － | X | X |
| ． | $\boldsymbol{\lambda}$ | ． | $\cdots$ | $\cdots$ | $\cdots$ | X | $\cdots$ | $\cdots$ | $\because$ | $\boldsymbol{\lambda}$ | X |  | N | X |
| $\because$ | $\because$ | $\ddot{\mathrm{X}}$ | $\because$ | $\because$ | $\ldots$ | $\cdots$ | $\because$ | $\ddot{X}$ | $\because$ | $\dot{\mathbf{x}}(\underline{2})$ | $\mathbf{x}$ | $\dot{x}$ | X |  |
| $\cdots$ | $\cdots$ | ． | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\ldots$ | ． | $\cdots$ |  | N | X | ． | $\cdots$ |
| $\ddot{\mathrm{X}}$ | $\ddot{\mathbf{x}}$ | X | x | $\because$ | $\cdots$ | $\ldots$ | $\because$ | $\dot{\mathrm{X}}(2)$ | $\ddot{\mathrm{x}}$ | X ${ }^{\text {2 }}$ | $\frac{\mathrm{X}}{\mathbf{X}}$ | X | $\ddot{\mathrm{x}}$ | X |
| ．． | ． |  | ． | ． | $\ldots$ | $\ldots$ | $\ldots$ |  | x | X | X | X | X | X |
| $\cdots$ | $\cdots$ | $x$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | ． | $\cdots$ | $\cdots$ | $x$ | J |  |  | ． |
|  | $\because$ | X | $\because$ | ． | $\cdots$ | X | $\because$ | $\because$ | $\because$ | X（5） | X | $\frac{\mathrm{x}}{\mathrm{x}(3)}$ | $\underset{\text { X }}{\text {（2）}}$ | $\cdots$ |
| X ${ }^{(2)}$ | $\because$ | X | $\cdots$ | $\ddot{\mathrm{x}}$ | － | X | $\because$ | $\cdots$ | $\dot{\mathrm{x}}$ | $\mathrm{X}(4)$ | X | X | X | $\ddot{\mathrm{x}}$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\because$ | $\cdots$ | ． | $\cdots$ | $\cdots$ | X | $\cdots$ | X | X | X | X |
| ． | ． | $\chi$ | $\cdots$ | $\lambda$ | $\lambda$ | $\cdots$ | ． | $\cdots$ | $\lambda$ | $\cdots$ | X | X | $\boldsymbol{\lambda}$ | ． |
| N | ． | $\cdots$ | x | X | $\cdots$ | $\cdots$ | X | X | X | X（3） | N | $\mathbf{~} \mathbf{( 2 )}$ | X | X ${ }^{(11)}$ |
| X | $\because$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |  |  | $\frac{\mathrm{N}}{\mathbf{N}}$ | $\frac{1}{\lambda}$ | $\hat{N}$ | $\frac{\lambda}{x}$ | － |
|  |  | $\dot{x}$ | $\because$ | $\cdots$ | $x$ | $x$ | $\because$ | ． | $\cdots$ |  | x | X |  |  |
| ． | x | X | ．$\cdot$ | ． | ． | ． | ． | ． | ． | X（4） | X（4） | X （9） | X（3） | $\ldots$ |
| ． | $\cdots$ | $\ldots$ | $\cdots$ | $\cdots$ | ．． | $\cdots$ | $\cdots$ | $\cdots$ | $x$ | $\underset{\text { X }}{\mathbf{X} 17}$ | $\underset{X}{X}$ | $\underline{X}$ | $\underline{\mathrm{x}}$（7） | $\cdots$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |  |  |  |  | － |  |
| －• | ． | $\cdots$ | $\cdots$ | ． | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | X | x | x | x | ． | ． |
| $\ddot{\mathrm{x}}$ | $\cdots$ | $\ddot{\mathrm{x}}$ | $\cdots$ | X | $\dot{\mathrm{x}}$ | $\dot{\mathbf{x}}(2)$ | $\ddot{\mathrm{x}}$ | ． | X | $\ddot{\mathrm{x}}$ | $\dot{\mathrm{x}}$ | $\dot{\text { x }}$ | $\dot{\text { x }}$ | $\ddot{\mathrm{x}}$ |
| ． | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | ． | $\cdots$ | X | $\cdots$ | $\cdots$ | $\cdots$ | X | $\cdots$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | ． | $\cdots$ | ．$\cdot$ | ． | $\cdots$ | ． | ． | $\cdots$ | ． | $\cdots$ | $\cdot$ |

In the following notes particulars are given regarding commodities included in each of the above index-numbers, but excluded from the preceding table for the reason already stated.

Fleetwood.-Cloth, Shoes, Floughs, Carts, Land, Horses, Cattle, Mules, Swine, Goats, Fowls, Rabbits, Pigeons, Wine, Ale, Beer, Spice, Wax, Figs, Charcoal.

## Jevons.-Tin Plates, Logwood.

Sauerbeck.-Petroleum, Nitrate of Soda.
Board of Trade.-Wine, Cotton Seed, Paraffin, Petroleum.
Laspeyres.-Logwood, Calfskin, Rags, Tar, Wine.
Paasche.-Mahogany, Calfskins, Bristles, Horse-hair, Wax, Quicksilver.
Conrad.-Mahogany, Hops, Calfskins, Bristles, Horse-hair, Wax, Quicksilver.
Soetbeer--Buckwheat, Oilcake, Veal, Calfskins, Horse-hair, Bristles, Bedfeathers, Bones, Buffalo Horns, Glue, Dried Prunes, Wine, Champagne, Allspice, Cassia Bark, Sago, Cochineal, Logwood, Rosewood, Mahogany, Rattan, Ivory, Quicksilver, Sulphur, Lime, Cement, Cordage, Rags, Guano, Gum-elastic, Resin, Pearl Ash, Pitch, Potash, Candles, Tar, Wax, Sewing Thread, Bottles, Sailcloth, Woollen Cloth, Flannels, Worsted, Carpets.

Prussian Government.-Lentils, Veal. 37 articles are given but only 15 are specified.

Hooker (Germany).—Cattle, Calves, Pigs (2), Hops, Petroleum.
Palgrave (France),-Oil-seed, Silk Stuffs, Gloves.
Falkner (France).-Beeves, Calves, Cows, Hogs, Sesamum, Lambskins, Kidskins, Silk Goods (2), Merinos, Blankets, Carpets, Tapestry, Gloves.

Hooker (France).-Cattle, Calves, Pigs, Wine, Nitrate of Soda.
Italy (Government).-Wine.
Walras (Switzerland).-Bread (2), Veal, Firewood (2).
Atkinson (India).-Mace, Millet Corn, Pulse, Fajra, other Grains, Ginger, Opium, Croton, Castor Oil, Dye, Bone Manure, Jute Goods, Silk Goods, Shellac.

Aldrich (U.S.A.).-Ship Bread (3), Boston Crackers (2), Oyster Crackers, Ship Biscuits, Soda Crackers, Dried Apples, Corn Meal, Ham, Lamb, Molasses (2), Nutmegs, Cornstarch (2), Blankets (2), Broadeloths (2), Calico, Carpets (3), Cassimeres (4), Checks, Horse Blankets, Print Cloths (2), Shawls, Sheetings, Shirtings, Ticking, Candles, Matches, Anvils, Butts, Door Knobs, Lead Shot, Locks (2), Meat Cutters, Cut Nails, Pocket Knives (25), Quicksilver, Rope (3), Saws (4), Scythes, Shovels, Wood Screws, Carbonate of Lead, Cement, Doors, Lime, Oxide of Zinc, Plate Glass (6), Putty, Tar, Turpentine, Window Glass, Alum, Potash, Vitriol, Brimstone, Calomel, Copperas, Flax Seed, Glycerine, Mercury, Muriatic Acid, Opium, Quinine, Soda Ash, Sugar of Lead (2), Sulphuric Acid, Furniture (3), Glassware (5), Pails (3), Tubs (4), Powder (2), Soap, Starch.

[^12]Coats (Canada)-Bran, Shorts, Turkeys, Chocolate, Cream of Tartar, Fresh Fruit (6), Honey, Maple Sugar, Oatmeal, Molasses, Tapioca, Vegetables (3), Camed Vegetables (3), Vinegar, Brass, Solder, Anvils, Axes, Grindstones, Hammers, Horseshoes, Mallets, Picks, Scrows, Soldering Irons, Vices, Coke, Carbide of Calcium, Matches, Hinges, Wire Nails, Cut Nails, Plaster of Paris, Sash Weights, Soil Pipe, Wire Cloth, Wire Fencing, Paints, Glass, Benzine, Glue, Boiled Oil, Putty, Paris Green. Shellac, Turpentine, Varnish, White Lead, Chairs, Tables, Sideboards, Bed Suites, Beds, Tumblers, Cups and Saucers, Toilet Sets, Dinner Sets, Knives, E.P. Knives and Forks, Wood Pails, Wood Tubs, Brooms, Alum, Bleaching Powder, Borax. Carbolic Acid, Caustic Soda, Copperas, Glycerine, Muriatic Acid, Opium, Quinino, Soda Ash, Sulphuric Acid, Furs (4), Binder Twine, Rope, Soap. Cattle, Beer, Shoes and Boots (3).

Australia (Wholesale).-Branbags, Cornsacks, Woolpacks, Leather (3), Bran, Pollard, Oatmeal, Ham, Honey, Macaroni, Sago, Mustard, Starch, Blue, Matches, Candles, Kerosene, Veal, Lamb, Cement, White Lead, Cream of Tartar, Sulphur.

Australia (Retail).-Bread, Sago, Jam, Oatmeal, Starch, Blue, Candles, Soap,
ns, Ham. Onions, Ham.

In addition to the authorities mentioned in the above table, investigations have also been made in the following countries, but details as to the commodities included therein are not available:-

| Country. | Name of Authority. | Years. | No. of Articles. |
| :---: | :---: | :---: | :---: |
| Great Britain- | Rice Vaughan Evelyn Mulhail | $\begin{gathered} 1675 \\ 1798 \\ 1854-1884 \end{gathered}$ | 50 |
| Germany (Hamburg)- | $\begin{array}{r} \text { Kral } \\ \text { Heinz } \\ \text { Schmitz } \end{array}$ | $\begin{aligned} & 1845-1884 \\ & 1850-1891 \\ & 1890-1910 \end{aligned}$ | $\begin{array}{r} 265 \\ 180 \\ 29 \end{array}$ |
| Prance- | D'Avenal <br> De Foville <br> Reforme Economique | $\begin{aligned} & 1200-1898 \\ & 1847-1880 \end{aligned}$ |  |
| Belgium- | Waxweiler | 1890-1910 |  |
| U.S. A- | Burchard <br> Falkner | $\begin{aligned} & 1825-1884 \\ & 1890-1899 \end{aligned}$ | 68 to 90 articles 90 articles |
| New Zealand- | McIlwraith | 1861-1910 | 33 to 45 articles |

An examination of the above statement clearly shews the great diversity in practice which existed in the selection of commodities in order to obtain the price data for the computation of Index-numbers. It may be seen that not one of the 67 commodities specified is common to all the Tndex-numbers. Several commoditios in ordinary use, such as coal, iron, cotton, wool, wheat, butter, etc., etc., are, however, common to the majority of the groups.

Applying the principles which have already been laid down in this Appendix for the selection of a group of commodities for the purpose of international comparisons the fellowing list has been compiled. Suggested " mass-units" (indicating relative consumption of each commodity in the corresponding unit of measurement) are also shewn in the following statement. These " mass-units" are based almost entirely on the Australian consumption, and are therefore suggested tentatively; they will probably require some amendment for international purposes.

## Proposed List of Commodities Suitable for Comparative Index-Numbers for the Western Nations with Mass Units.

| Commodities, | Unit of Measurement. | Mass. unit. | Commodities, | Unit of Measure ment. | Massunit. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Group I. |  |  | Grour V. |  |  |
| Metals and Coal. |  |  | Groceries, Etc. |  |  |
| Copper ${ }^{\text {. }}$ | ton. | 1 | Cocoa Beans . | 1 lb . | 100 |
| Iron, Pig | ton ${ }^{\text {- }}$ | 28 | Coffee Beans . . | 1 l . | 200 |
| Lead, Pig | ton | 1 | Currants | fb. | I, 400 |
| Silver, Bars | Oz. | 20,000 | Flour (Wheat) . . | ton | 48 |
| Tin, Block | ton | $\frac{1}{4}$ | Flour (Rye) . | ton | 10 |
| Zinc . | ton | 1 | Hops . . | 1 l. | 120 |
| Coal | ton | 600 | Malt | cwt. | 10 |
|  |  |  | Pepper . . . | 1 l . | 80 |
| Group IL. |  |  | Raisins . . . | lb. | 1,400 |
| Textiles, Leather, |  |  | Sago | lb. | 800 |
| Erc. |  |  | Salt | ton | 8 |
| Hides | cach | 120 | Sugar | ton | 22 3.000 |
| Sheep Skins | each | 400 | Tobacco . | lb. | 3,300 |
| Cotton, Raw | lb. | 24,000 | Candles . | 1 l. | 1,600 |
| Flax | cwt. | 18 | Mustard | Ib. | 72 |
| Hemp | ewt. | 18 | Kerosene.. | gallon | 1,700 |
| Jute | cwt. | 40 | Kerosene.. | gallon | 1,700 |
| Silk | lb. | 250 |  |  |  |
| Wool | 1b. | 12,200 | $\\| \text { Meat, Erc. }$ |  |  |
|  |  |  | Beef | 1 b . | 39,000 |
| Group ILI. |  |  | Veal $\quad$. | Ib. | 2,000 |
| Agricultural Pro- |  |  | Mutton . | Ib. | 33,000 |
| DUCE. |  |  | Pork | lb. | 3,700 |
| Barley | bushel | 250 | Fish | lb. | 2,400 |
| Beans | bushel | 50 | Lard | lb. | 200 |
| Hay | ton | 270 | Tallow | cwt. | 35 |
| Maize | bushel | 1,000 |  |  |  |
| Oats | bushel | 1,300 | Group VII. |  |  |
| Peas | bushel | 55 | Building Materials |  |  |
| Rice | ton | 2 | Bricks . . . | per 1,000 | 50 |
| Rye | bushel | 100 | Timber . . | $100 \mathrm{~s} . \mathrm{ft}$. | 350 |
| Straw | ton | 25 | Cement . . | cask | 30 |
| Wheat | bushel | 500 | Lime . | ton | 10 |
| Potatoes | ton | 40 | Slates | per 1,000 | $\underline{2}$ |
| Group IV. |  |  | Group Vitit. |  |  |
| Dairy Produce |  |  | Miscellaneous. |  |  |
| Bacon | lb. | 3,200 | Caoutchouc (Raw |  |  |
| Butter | lb. | 9,500 | Rubber) . . | cwt. | 50 |
| Cheese | lb. | 1,500 | Soda Carbonate | ton | $\frac{1}{2}$ |
| Eggs | doz. | 1,800 | Saltpetre . . | ton | ${ }_{2}^{10}$ |
| Milk | quart | 30,000 | Sulphter | ton | $\frac{1}{4}$ |
| Honey | lb. | 600 | Cream of Tartar | lb. | $40{ }^{3}$ |

Summary of Conclusions.-The conclusion of the whole matter divides itself into two heads, viz., (i.) that which concerns the list of commodities, the number of units to be taken, and changes in this list ; and (ii). that which concerns the technique of computing the price-index.
I. (i.) The list should contain (a) commodities easily identifiable as to character and quality ; (b) commodities for which there are world markets. Commodities for which only a local market exists should constitute a separate list for local purposes.
(ii.) The number of units taken should represent the average usage among all the nations included in the comparison.
(iii.) The number of commodities and the units assigned should be subject to decennial revision.
(iv.) During each decennium, the series of units and commodities used must necessarily be those ascertained for the preceding decennium.
(v.) At the close of each decennium it is desirable that the price-indexes found for it should be revised on the introduction of the next decennial list of commodities, and the units of usage assigned to them.
(vi.) In order that the price-indexes, while substantially accurate from the standpoint of gold exchange-value, shall yet represent the actual usage of mankind in respect to commodities, its basis, owing to change of normal regimen, should be subject to continuous modifications.
(vii.) This is practically secured by varying the regimen units of commodities yearly one-tenth of the decennial difference, the control of the number of units assigned being properly attended to.
(viii.) Subdivisions of the list of commodities should be so made that the items within a subdivision are homogeneous with respect to the ratio of the value of the raw material to the value of the labour in the finished product.
(ix.) There can be no really perfect continuity between the price-indexes for periods characterised by different regimens.
(x.) Since economic inquiries of an exact character must take account of variations in the relative usage of commodities, comparisons between widely different periods must take account not only of variations in the exchange-value of gold, but also in average regimen.

In regard to technique, the common-sense method of adopting, for the purposes of comparison, a series of units of definite commodities and finding the aggregate of expenditure according to these, is unquestionably the best method of tracing the variations in the exchange-value of gold against commodities. The matter may be summed up as follows :-
11. (i.) For initial comparisons, the experience of each decennium will furnish the units that are used for the following decenniums.
(ii.) The method of finding the ratio of aggregate expenditures is not only the simplest but the best.
(iii.) Price-ratios are not satisfactory unless the weighted geometric mean be found, and using for the weights assigned the mean expenditure for any two periods. The method then becomes sensibly identical with the aggregate expenditure method, but the arithmetical work involved is prohibitive, and the method is not suited for continuous records.
(iv.) Although the apparent generality of the price-ratio method is not wholly an illusion, it practically has no advantages whatever over the aggregate of expenditure method, the latter being arithmetically very simple.
(v.) With the aggregate of expenditure method, the influence of any umcertainty in the series of commodities or in their prices, on the priceindex deduced, can more readily be seen than with the price-ratio method.
(vi.) The establishment of an international series of commodities would have for its immediate object the comparison of the exchange-value of the gold-unit in each nation on the basis of a common average regimen.
(vii.) This may not be the best system of units for the nation itself.
(viii.) Each nation may find it necessary, therefore, to have also its own list, and its own units, and to deduce price-indexes representing the variation of the exchange-value of gold so far as the nation itself is concerned in its internal relations.
(ix.) In general the fluctuations on the two bases will not be quite identical, the difference being due to what may be called change of regimen.
(x.) Experience may, however, shew that the relation between the two can be readily determined, or is a negligible quantity, so that ultimately one list may suffice.

Regarding general matters the following may be said :-
III. (i.) It may, on first consideration, appear unsatisfactory that through long intervals of time the same class of commodities cannot be utilised for determining absolutely variations in the exchange-value of gold. If, however, the method involving slow variations of regimen be followed, there is no strong objection to the method indicated in this paper.
(ii.) Per contra, it is to be preferred, since it applies to the existing regimen at all points of time, at least when corrected as indicated by continuously varying the regimen.
(iii.) By these methods a satisfactory kind of continuity can be secured, which although only a pseudo-continuity as regards the exchange-value of gold, is nevertheless a real continuity as regards the usage of gold in relation to all other commodities on the list.
(iv.) It is therefore of much greater value than would be furnished by priceindexes based, if it were possible-which it is not-on a continued use of the commodities of the past as the basis of determination.
(v.) The method of a slowly changing " commodity unit," though establishing theoretically only approximate values, nevertheless yields results which more truly represent the aggregate of the facts, than does the method of absolute comparisons based upon the same number of units and the same list of commodities.
(vi.) Special investigations may nevertheless be considered necessary between any two years for any definite series of commodities, and any definite number of units in connection therewith.


[^0]:    * First 9 months.

[^1]:    * Average prices for first 9 months only.

[^2]:    * For the flrst 0 months. $\quad t$ The rents are shewn to the nearest pemny.

[^3]:    * Forasmuch as the money-unit constitutes a unique common measure of exchange-values.

[^4]:    * The consumption per head per annum is about 32 loaves of bread, 3 lbs. of tea, and 16 quarts of milk
    $t$ Here it may be mentioned that computed from the geometric mean of the price-ratios, weighted according to the arithmetic mean of the welghts, we should obtain 109.53 . See next section.
    $\ddagger$ This method is wholly unsatisfactory.

[^5]:    * It is of course evident that if this can be done it is also possible to work with the relative units used of the various commodities; thus formula (2) is more convenient. It is also to be preferred in every way as will hereafter be shewn.
    $\dagger$ As given by formula (2) since the units are identical.

[^6]:    * By formula (2), viz., the ratio of the aggregate expenditures, we get 109.47; using arithmetic-mean weights and formula (11) we get 109.53.)
    $t$ It is shewn hereinafter that formule (11) and (2) are sensibly identical when the weights and units are properly determined.

[^7]:    * The question of variation of regimen I have considered elsewhere, but not herein.
    $t$ This has been done elsewhere.

[^8]:    * For so long as the unintelligence and bad-will of mankind necessitates so wasteful a procedure, the commodity gola (and silver) may be regarded as in some way the real basis of the idea of money, and this notwithstauding the fact that the use of the precious metals will probably be greatly limited or may even cease when international obligations are certain to be honoured, or when an international credit system is sufficiently assured; a consummation which doubtless will tend to be reached in proportion as the jeopardy of war is diminished. So long as we bear in mind that we are thinking of money in general, rather than actual gola, we may use the expression "purchasing-efficiency or exchange-value of gold" to represent that reciprocal of the relation between the commodity gold and any other commodity, which is expressed as price. In this view "price" is looked upon as defining the instantaneous potentiality of exchange, by the artiffce of a supposed real commodity, viz., gold.

[^9]:    * Contrast such forms of steel mannfacture as heavy springs for railway fittings, with watchsprings; or contrast say the production of heavy and chiffon silks. $\dagger$ For example with such commodities as watch-springs, in which the value of the raw material is wholly negligible, the resulting price depends practically wholly upon the cost of labour directly or indirectly.

[^10]:    * To revert to a former illustration, the value of a watch-spring may be said to bo due wholly to the cost of habour required to produce it, and it stands, therefore, in a very different economic position to, sny a large and simple casting, the raw materinl being pig-iron, because for the production of the latter the element of cost of labour enters relatively to a much less extent. And even if in the last analysis it could be assumed that the original raw materinh is withont value until labour is expended thoreon, which is not always true, the fact still remains that we shall do well in any classiflcation to have vegard to the value of labour in production, as compared with the value of the raw material.

[^11]:    * This, as has been shewn, gives in general almost the same results as the geometric mean.

[^12]:    W. Bureau of Labor (U.S.A.).-Canned Corn, Canned Peas, Canned Tomatoes, Dried Apples, Prunes, Glucose, Corn Meal (2), Molasses, Fresh Vegetables, Onions, Broadcloth, Drill, Gingham, Horse Blankets, Hosiery, Overcoatings, Sheetings, Shirtings, Tickings, Underwear, Sicilian Cloth, Cashmere, Poplar, Panama, Worsted and Worsted Yarn, Candles, Matches, Augers, Axes, Barb Wire, Butts, Chisels, Copper Wire, Steel Door Knobs, Files, Hammers, Lead Pipe, Locks, Cut Nails, Wire Nails, Planes, Saws (2), Shovels, Steel Billets, Steel Rails, Steel Sheets, Trowels, Vises, Wood Screws, White Lead, Cement (2), Doors, Lime, Oxide of Zinc, Plate Glass, Putty, Resin, Shingles, Turpentine, Window Glass, Alum, Brimstone, Glycerine, Muriatic Acid, Opium, Quinine, Sulphuric Acid, Earthenware Plates, Cups and Saucers, Bed Sets, Chairs (2), Tables, Glassware (3), Cutlery, Woodenware, Cotton Seed, Meal, Newspaper, Wrapping Paper, Rope, Soap, Cattle, Fowls, Horses, Mules, Swine, Bread, Blankets, Carpets, Shoes and Boots (4), Quicksilver.

